

HI EVERYONE,

THE REAL LEARNING IN MATHEMATICS HAPPENS WHEN YOU ACTIVELY ENGAGE WITH A PROBLEM, EXPLORE DIFFERENT METHODS, AND WORK THROUGH CHALLENGES. THEREFORE, WE STRONGLY ENCOURAGE YOU TO USE THIS SOLUTION KEY RESPONSIBLY.

PLEASE ATTEMPT ALL THE PROBLEMS ON YOUR OWN FIRST, GIVING THEM YOUR BEST AND MOST HONEST EFFORT. THESE SOLUTIONS ARE TO HELP YOU GET UNSTUCK ON A PROBLEM AFTER YOU HAVE ALREADY TRIED YOUR BEST.

YOUR EFFORT AND DEDICATION ARE THE TRUE KEYS TO SUCCESS.

**Exam Date: 23rd November 2025**

Syllabus: Sequence and Series

Sub: Mathematics

**CT-07 JEE Advanced Solution**

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### Section 1: Single Correct Answer Type

1. The sum of the first 9 terms of the series  $\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \dots$  is equal to:

(A) 71

(B) 96

(C) 142

(D) 192

**Answer: (B)**

**Solution:** Let the  $n^{\text{th}}$  term of the series be  $T_n$ . The numerator is the sum of cubes:  $\sum k^3 = \left[\frac{n(n+1)}{2}\right]^2$ . The denominator is the sum of odd numbers:  $n^2$ .

$$T_n = \frac{\left[\frac{n(n+1)}{2}\right]^2}{n^2}$$
$$T_n = \frac{n^2(n+1)^2}{4n^2} = \frac{(n+1)^2}{4}$$

We need the sum of the first 9 terms ( $n = 1$  to 9):

$$S_9 = \sum_{n=1}^9 T_n = \frac{1}{4} \sum_{n=1}^9 (n+1)^2$$
$$S_9 = \frac{1}{4} [2^2 + 3^2 + \dots + 10^2]$$

Using  $\sum_{k=1}^{10} k^2 = 385$ :

$$S_9 = \frac{1}{4}(385 - 1^2)$$
$$S_9 = \frac{384}{4} = 96$$

2. The sum of the infinite series  $\frac{3}{11} + \frac{33}{11^2} + \frac{333}{11^3} + \dots$  is equal to:

(A)  $\frac{30}{10}$

(B)  $\frac{33}{10}$

(C)  $\frac{33}{100}$

(D)  $\frac{11}{3}$

**Answer: (B)**

**Solution:**

$$S = \frac{3}{11} + \frac{33}{11^2} + \frac{333}{11^3} + \dots$$
$$S = \frac{1}{3} \sum_{n=1}^{\infty} \frac{10^n - 1}{11^n}$$

$$S = \frac{1}{3} \left[ \sum \left( \frac{10}{11} \right)^n - \sum \left( \frac{1}{11} \right)^n \right]$$

Calculating the two infinite GP sums:

$$S_1 = \frac{10/11}{1 - 10/11} = 10$$

$$S_2 = \frac{1/11}{1 - 1/11} = \frac{1}{10}$$

Substitute back:

$$S = \frac{1}{3} \left( 10 - \frac{1}{10} \right) = \frac{33}{10}$$

## Section 2: Multiple Correct Answer Type

3. The  $2^{nd}$ ,  $5^{th}$ , and  $9^{th}$  terms of a non-constant Arithmetic Progression are in Geometric Progression. Let  $a$  be the first term and  $d$  be the common difference of the A.P. Which of the following is/are correct?
- (A) The common ratio of the G.P. is  $\frac{4}{3}$ .  
 (B) The ratio  $a : d$  is equal to  $8 : 1$ .  
 (C) The sum of the first 3 terms of the A.P. is  $27d$ .  
 (D) The  $4^{th}$ ,  $10^{th}$ , and  $16^{th}$  terms of the A.P. are also in G.P.

**Answer: (A), (B), (C)**

**Solution:** Given  $a + d, a + 4d, a + 8d$  are in GP.

$$\begin{aligned} (a + 4d)^2 &= (a + d)(a + 8d) \\ a^2 + 8ad + 16d^2 &= a^2 + 9ad + 8d^2 \\ 8d^2 = ad &\implies a = 8d \quad (\because d \neq 0) \end{aligned}$$

(A) Common Ratio  $r$ :

$$r = \frac{a + 4d}{a + d} = \frac{12d}{9d} = \frac{4}{3} \quad (\text{Correct})$$

(B) Ratio  $a : d = 8 : 1$ . (Correct)

(C) Sum of first 3 terms:

$$S_3 = 3a + 3d = 3(8d) + 3d = 27d \quad (\text{Correct})$$

(D) Terms  $a + 3d, a + 9d, a + 15d$  become  $11d, 17d, 23d$ .

$$(17d)^2 \neq (11d)(23d) \quad (\text{Incorrect})$$

4. Let a G.P. have  $2n$  terms. The sum of the odd-numbered terms is  $85$  and the sum of the even-numbered terms is  $170$ . Which of the following is/are true?
- (A) The common ratio is  $2$ .  
 (B) The sum of all terms is  $255$ .  
 (C) If the first term is  $1$ , then  $n = 4$ .  
 (D) If the first term is  $5$ , then  $n = 5$ .

**Answer: (A), (B), (C)**

**Solution:**

$$\begin{aligned} S_{\text{even}} &= r \cdot S_{\text{odd}} \\ 170 &= r \cdot 85 \implies r = 2 \quad (\text{Option A Correct}) \end{aligned}$$

Sum of all terms:

$$S_{\text{total}} = 85 + 170 = 255 \quad (\text{Option B Correct})$$

Using sum formula  $S_{2n} = \frac{a(r^{2n}-1)}{r-1}$ :

$$255 = a(2^{2n} - 1)$$

If  $a = 1$ :

$$255 = 2^{2n} - 1 \implies 2^{2n} = 256 \implies 2n = 8 \implies n = 4 \quad (\text{Option C Correct})$$

If  $a = 5$ :

$$51 = 2^{2n} - 1 \implies 2^{2n} = 52 \quad (\text{No integer } n)$$

### Section 3: Matrix Match Type

5. Match the problem descriptions in Column-I with their correct values in Column-II.

Column-I	Column-II	Values
(A) The minimum value of $m$ for which the numbers $5^{1+x} + 5^{1-x}$ , $m/2$ , and $25^x + 25^{-x}$ form an A.P. for some $x \in \mathbb{R}$	(P)	1
(B) The maximum value of $(a+b)(c+d)$ if $a, b, c, d$ are positive real numbers and $a+b+c+d=2$ .	(Q)	12
(C) The value of $1/\alpha$ if $\alpha$ is the least positive number such that $4\alpha x^2 + \frac{1}{x} \geq 1$ for all $x > 0$ .	(R)	27
(D) The maximum value of $abc$ if $a, b, c > 0$ and $a+b+c=3$ .	(S)	4

**Answer: A-Q, B-P, C-R, D-P**

**Solution:** (A) Condition for AP:  $2(\text{middle}) = \text{first} + \text{last}$ .

$$m = (5^{1+x} + 5^{1-x}) + (25^x + 25^{-x})$$

$$m = 5(5^x + 1/5^x) + ((5^x)^2 + 1/(5^x)^2)$$

Using  $t + 1/t \geq 2$  for  $t > 0$ :

$$m \geq 5(2) + 2 = 12 \implies m_{\min} = 12 \quad (\text{Matches Q})$$

(B) Let  $X = a + b$  and  $Y = c + d$ . Given  $X + Y = 2$ . Maximize  $XY$ . Product is max when  $X = Y$ .

$$X = Y = 1 \implies \text{Max value} = 1 \cdot 1 = 1 \quad (\text{Matches P})$$

(C)  $f(x) = 4\alpha x^2 + \frac{1}{x} = 4\alpha x^2 + \frac{1}{2x} + \frac{1}{2x}$ . By AM-GM inequality:

$$\frac{4\alpha x^2 + \frac{1}{2x} + \frac{1}{2x}}{3} \geq \sqrt[3]{4\alpha x^2 \cdot \frac{1}{4x^2}} = \sqrt[3]{\alpha}$$

$$\text{Given sum} \geq 1 \implies 3\sqrt[3]{\alpha} \geq 1$$

$$27\alpha \geq 1 \implies \alpha \geq 1/27$$

$$\therefore 1/\alpha = 27 \quad (\text{Matches R})$$

(D) Maximize  $abc$  subject to  $a + b + c = 3$ .

$$\frac{a+b+c}{3} \geq \sqrt[3]{abc}$$

$$1 \geq \sqrt[3]{abc} \implies abc \leq 1 \quad (\text{Matches P})$$

6. Match the Series/Expressions in Column-I with their evaluations in Column-II.

Column-I	Column-II	Values
(A) The value of $\frac{S_3(1+8S_1)}{S_2^2}$ where $S_k = \sum n^k$ .	(P)	90
(B) A.P. Sum ratio $\frac{S_7}{S_{11}} = \frac{6}{11}$ . Terms distinct natural numbers. $130 < T_7 < 140$ . Find common difference.	(Q)	9
(C) A.P. even terms. $S_{odd} = 24, S_{even} = 30$ . $Last - First = 10.5$ . Find n.	(R)	8
(D) The value of $\sum_{n=1}^4 (3^n - 2^n)$ .	(S)	1

**Answer: A-Q, B-Q, C-R, D-P**

**Solution: (A)** Using formulas  $S_3 = S_1^2$  and  $1 + 8S_1 = (2n + 1)^2$ .

$$\begin{aligned} \text{Exp} &= \frac{S_1^2(2n+1)^2}{S_2^2} = \left( \frac{S_1(2n+1)}{S_2} \right)^2 \\ &= \left( \frac{\frac{n(n+1)}{2}(2n+1)}{\frac{n(n+1)(2n+1)}{6}} \right)^2 = \left( \frac{6}{2} \right)^2 = 9 \quad \text{(Matches Q)} \end{aligned}$$

**(B)** Ratio of sums:

$$\begin{aligned} \frac{S_7}{S_{11}} &= \frac{7(2a+6d)}{11(2a+10d)} = \frac{6}{11} \implies 7(2a+6d) = 6(2a+10d) \\ 14a + 42d &= 12a + 60d \implies 2a = 18d \implies a = 9d \end{aligned}$$

Given  $130 < T_7 < 140$  where  $T_7 = a + 6d = 15d$ :

$$130 < 15d < 140 \implies 8.6 < d < 9.3$$

Since terms are natural numbers,  $d$  must be integer  $\implies d = 9$ . **(Matches Q)**

**(C)** System of equations for Even/Odd sums:

$$\begin{aligned} S_{even} - S_{odd} &= (a_2 - a_1) + \dots + (a_{2n} - a_{2n-1}) = \frac{n}{2}d = 6 \implies nd = 12 \\ l - a &= (n-1)d = 10.5 \implies nd - d = 10.5 \\ 12 - d &= 10.5 \implies d = 1.5 \\ n &= 12/1.5 = 8 \quad \text{(Matches R)} \end{aligned}$$

**(D)** Calculation:

$$\begin{aligned} \sum_{n=1}^4 (3^n - 2^n) &= (3 + 9 + 27 + 81) - (2 + 4 + 8 + 16) \\ &= 120 - 30 = 90 \quad \text{(Matches P)} \end{aligned}$$

## Section 4: Paragraph Type

7. The value of the infinite series  $1 + \frac{4}{5} + \frac{9}{25} + \frac{16}{125} + \frac{25}{625} + \dots$  is equal to:

(A)  $\frac{75}{32}$

(B)  $\frac{25}{8}$

(C)  $\frac{125}{64}$

(D)  $\frac{5}{2}$

**Answer: (A)**

**Solution:** Let  $S = \sum_{n=1}^{\infty} \frac{n^2}{5^{n-1}}$ .

$$\begin{aligned} S &= 1 + \frac{4}{5} + \frac{9}{25} + \dots \\ \frac{1}{5}S &= \frac{1}{5} + \frac{4}{25} + \dots \\ \frac{4}{5}S &= 1 + \frac{3}{5} + \frac{5}{25} + \dots \quad \text{(Difference of terms)} \end{aligned}$$

Let  $S' = \frac{4}{5}S$ . Perform AGP subtraction again:

$$\begin{aligned}
 S' &= 1 + \frac{3}{5} + \frac{5}{25} + \dots \\
 \frac{1}{5}S' &= \frac{1}{5} + \frac{3}{25} + \dots \\
 \frac{4}{5}S' &= 1 + \frac{2}{5} + \frac{2}{25} + \dots = 1 + \frac{2/5}{4/5} = 1.5 \\
 S' &= 1.5 \times \frac{5}{4} = \frac{15}{8} \\
 S &= \frac{15}{8} \times \frac{5}{4} = \frac{75}{32}
 \end{aligned}$$

8. The value of the sum  $\sum_{k=1}^{10} (k+1)(k+2)$  is equal to:

(A) 505

(B) 570

(C) 440

(D) 650

**Answer: (B)**

**Solution:**

$$\begin{aligned}
 \text{Sum} &= \sum_{k=1}^{10} (k^2 + 3k + 2) \\
 &= \sum k^2 + 3 \sum k + \sum 2 \\
 &= 385 + 3(55) + 20 \\
 &= 385 + 165 + 20 = 570
 \end{aligned}$$

## Section 5: Integer Answer Type

9. If  $S = 10^9 + 2 \cdot 11 \cdot 10^8 + 3 \cdot 11^2 \cdot 10^7 + \dots + 10 \cdot 11^9 = k \cdot 10^9$  then find the value of k.

**Answer: 100**

**Solution:** Rearrange  $S$ :

$$S = 10^9 \sum_{j=0}^9 (j+1) \left(\frac{11}{10}\right)^j$$

Let  $x = 1.1$ . Sum is  $A = 1 + 2x + \dots + 10x^9$ .

$$\begin{aligned}
 (1-x)A &= (1+x+\dots+x^9) - 10x^{10} \\
 -0.1A &= \frac{x^{10}-1}{x-1} - 10x^{10} \\
 -0.1A &= 10(x^{10}-1) - 10x^{10} = -10 \\
 A &= 100
 \end{aligned}$$

Thus  $S = 10^9 \times 100 \implies k = 100$ .

10. Let  $a_1, a_2, \dots, a_{49}$  be in A.P. such that  $\sum_{k=0}^{12} a_{4k+1} = 416$  and  $a_9 + a_{43} = 66$ . If  $a_1^2 + a_2^2 + \dots + a_{17}^2 = 140m$ , find the value of m.

**Answer: 34**

**Solution:** The sum is of 13 terms:  $a_1, a_5, \dots, a_{49}$ .

$$\frac{13}{2}(a_1 + a_{49}) = 416 \implies a_1 + a_{49} = 64$$
$$2a_1 + 48d = 64$$

Given  $a_9 + a_{43} = 66$ :

$$(a_1 + 8d) + (a_1 + 42d) = 66 \implies 2a_1 + 50d = 66$$

Solving the two equations:

$$2d = 2 \implies d = 1$$
$$2a_1 + 48 = 64 \implies a_1 = 8$$

Calculate required sum  $S'$ :

$$S' = \sum_{k=1}^{17} (8 + (k-1)d)^2 = \sum_{j=8}^{24} j^2$$
$$= \sum_{j=1}^{24} j^2 - \sum_{j=1}^7 j^2$$
$$= \frac{24(25)(49)}{6} - \frac{7(8)(15)}{6}$$
$$= 4900 - 140 = 4760$$

Finally solve for  $m$ :

$$140m = 4760 \implies m = 34$$