$$\boxed{\text{Exam Date: 16th November 2025}}$$

Syllabus: Sequences & Series

Sub: Mathematics $\qquad$ **CT-09 JEE Main Regular - Solution** $\qquad$ Prof. Chetan Sir

1. The product $2^{\frac{1}{4}} \cdot 4^{\frac{1}{16}} \cdot 8^{\frac{1}{48}} \cdot 16^{\frac{1}{128}} \cdot \ldots$ to ∞ is equal to:

(A) 2 $\qquad$ (B) $2^{\frac{1}{2}}$ $\qquad$ (C) 1 $\qquad$ (D) $2^{\frac{1}{4}}$

Answer: (B)
Solution: Let the product be P. First, we express all bases as powers of 2.

$$P = (2^1)^{\frac{1}{4}} \cdot (2^2)^{\frac{1}{16}} \cdot (2^3)^{\frac{1}{48}} \cdot (2^4)^{\frac{1}{128}} \cdot \ldots$$
$$P = 2^{\frac{1}{4}} \cdot 2^{\frac{2}{16}} \cdot 2^{\frac{3}{48}} \cdot 2^{\frac{4}{128}} \cdot \ldots$$
$$P = 2^{\frac{1}{4}} \cdot 2^{\frac{1}{8}} \cdot 2^{\frac{1}{16}} \cdot 2^{\frac{1}{32}} \cdot \ldots$$

When multiplying powers with the same base, we add the exponents.

$$P = 2^S \quad \text{where } S = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \ldots$$

The series S is an infinite Geometric Progression (G.P.) with: First term $a = \frac{1}{4}$ Common ratio $r = \frac{1/8}{1/4} = \frac{1}{2}$ The sum of an infinite G.P. is $S_\infty = \frac{a}{1-r}$, for $|r| < 1$.

$$S = \frac{1/4}{1 - 1/2} = \frac{1/4}{1/2} = \frac{1}{4} \times 2 = \frac{1}{2}$$

Therefore, the product $P = 2^S = 2^{\frac{1}{2}}$.

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2. If $a_1, a_2, a_3, ..., a_n$ are in A. P. and $a_1 + a_4 + a_7 + \cdots + a_{16} = 114$, then $a_1 + a_6 + a_{11} + a_{16}$ is equal to:

(A) 64 $\qquad$ (B) 98 $\qquad$ (C) 38 $\qquad$ (D) 76

Answer: (D)
Solution: Let the first term be a_1 and the common difference be d. The given sum is $S_1 = a_1 + a_4 + a_7 + a_{10} + a_{13} + a_{16} = 114$. The indices $1, 4, 7, 10, 13, 16$ form an A.P. with 6 terms. Express each term in S_1 in terms of a_1 and d:

$$S_1 = (a_1) + (a_1 + 3d) + (a_1 + 6d) + (a_1 + 9d) + (a_1 + 12d) + (a_1 + 15d) = 114$$

Combine the terms:

$$6a_1 + (3 + 6 + 9 + 12 + 15)d = 114$$
$$6a_1 + 45d = 114$$

Divide the entire equation by 3:

$$2a_1 + 15d = 38$$

Now, let's analyze the sum we need to find, $S_2 = a_1 + a_6 + a_{11} + a_{16}$. Express these terms in terms of a_1 and d:

$$S_2 = (a_1) + (a_1 + 5d) + (a_1 + 10d) + (a_1 + 15d)$$

Combine the terms:

$$S_2 = 4a_1 + (5 + 10 + 15)d = 4a_1 + 30d$$

We can factor out a 2 from this expression:

$$S_2 = 2(2a_1 + 15d)$$

We already found that $2a_1 + 15d = 38$.

$$S_2 = 2(38) = 76$$

3. The number of terms common to the two A.P.'s 3, 7, 11, ..., 407 and 2, 9, 16, ..., 709 is

Answer: 14
Solution:
We are given two arithmetic progressions:
First A.P. (AP_1): $3, 7, 11, \ldots, 407$
First term $a_1 = 3$, common difference $d_1 = 4$.
$Second A.P.$ (AP$_2$): $2, 9, 16, \ldots, 709$
First term $b_1 = 2$, common difference $d_2 = 7$.

Step 1: Find the first common term

We list the terms of each A.P. to find the first match.
$$AP_1 : 3, 7, 11, 15, 19, \mathbf{23}, 27, \ldots$$
$$AP_2 : 2, 9, 16, \mathbf{23}, 30, \ldots$$
The first common term is 23. Let $c_1 = 23$.

Step 2: Find the common difference of the common A.P.

The common difference d of the A.P. of common terms is
the Least Common Multiple (LCM) of d_1 and d_2.
$$d = \text{LCM}(d_1, d_2)$$
$$d = \text{LCM}(4, 7)$$
$$d = 28$$
So, the A.P. of common terms is $23, 51, 79, \ldots$

Step 3: Find the number of common terms

The common terms must be $\leq \min(407, 709)$.
The last possible term for the common A.P. is $L = 407$.
We use the formula for the number of terms k :
$$k = \left\lfloor \frac{L - a}{d} \right\rfloor + 1$$
Substituting $L = 407, a = 23,$ and $d = 28$:
$$k = \left\lfloor \frac{407 - 23}{28} \right\rfloor + 1$$
$$k = \left\lfloor \frac{384}{28} \right\rfloor + 1$$
$$k = \left\lfloor \frac{96}{7} \right\rfloor + 1$$
Since $\frac{96}{7} \approx 13.71,$ the greatest integer is $\lfloor 13.71 \ldots \rfloor = 13$.
$$k = 13 + 1$$
$$k = 14$$

Thus, there are 14 common terms.

4. Let S_n denote the sum of the first n terms of an A. P.. If $S_4 = 16$ and $S_6 = -48$, then S_{10} is equal to:

(A) -320   (B) -380   (C) -260   (D) -410

Answer: (A)
Solution: We use the sum formula $S_n = \frac{n}{2}[2a + (n-1)d]$. Given $S_4 = 16$:

$$\frac{4}{2}[2a + (4-1)d] = 16$$
$$2(2a + 3d) = 16$$
$$2a + 3d = 8 \quad \cdots (1)$$

Given $S_6 = -48$:

$$\frac{6}{2}[2a + (6-1)d] = -48$$
$$3(2a + 5d) = -48$$
$$2a + 5d = -16 \quad \cdots (2)$$

Now we solve the system of linear equations. Subtract (1) from (2):

$$(2a + 5d) - (2a + 3d) = -16 - 8$$
$$2d = -24$$
$$d = -12$$

Substitute $d = -12$ into (1):

$$2a + 3(-12) = 8$$
$$2a - 36 = 8$$
$$2a = 44$$
$$a = 22$$

We need to find S_{10}:

$$S_{10} = \frac{10}{2}[2a + (10-1)d]$$
$$S_{10} = 5[2(22) + 9(-12)]$$
$$S_{10} = 5[44 - 108]$$
$$S_{10} = 5[-64] = -320$$

5. Let $x, y > 0$. If $x^3 y^2 = 2^{15}$, then the least value of $3x + 2y$ is

(A) 30   (B) 32   (C) 36   (D) 40

Answer: (D)
Solution: We need to find the minimum value of $3x + 2y$ using the A.M.-G.M. inequality. The G.M. will involve the term $x^3 y^2$. To get this, we need to split $3x$ into $x + x + x$ and $2y$ into $y + y$. We apply A.M.-G.M. to the 5 positive numbers x, x, x, y, y.

$$\text{A.M.} = \frac{x + x + x + y + y}{5} = \frac{3x + 2y}{5}$$
$$\text{G.M.} = \sqrt[5]{x \cdot x \cdot x \cdot y \cdot y} = \sqrt[5]{x^3 y^2}$$

By the A.M.-G.M. inequality, A.M. $\geq$ G.M.:

$$\frac{3x + 2y}{5} \geq \sqrt[5]{x^3 y^2}$$

We are given $x^3 y^2 = 2^{15}$.

$$\frac{3x + 2y}{5} \geq \sqrt[5]{2^{15}}$$

$$\frac{3x + 2y}{5} \geq (2^{15})^{1/5}$$
$$\frac{3x + 2y}{5} \geq 2^3$$
$$\frac{3x + 2y}{5} \geq 8$$
$$3x + 2y \geq 40$$

The least value of $3x + 2y$ is 40.

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6. The sum of an infinite geometric series with positive terms is 3 and the sum of the cubes of its terms is $\frac{27}{19}$. Then the common ratio of this series is:

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{2}{9}$ (D) $\frac{4}{9}$

Answer: (B)
Solution: Let the series be $a, ar, ar^2, \ldots$. Given $S_\infty = 3$. The formula is $S_\infty = \frac{a}{1-r}$.

$$\frac{a}{1-r} = 3 \implies a = 3(1-r) \quad \cdots (1)$$

The series of cubes is $a^3, (ar)^3, (ar^2)^3, \ldots$ which is $a^3, a^3 r^3, a^3 r^6, \ldots$. This is a new G.P. with first term $A = a^3$ and common ratio $R = r^3$. The sum of this new series is $\frac{27}{19}$.

$$\frac{a^3}{1-r^3} = \frac{27}{19} \quad \cdots (2)$$

Substitute (1) into (2):

$$\frac{[3(1-r)]^3}{1-r^3} = \frac{27}{19}$$
$$\frac{27(1-r)^3}{(1-r)(1+r+r^2)} = \frac{27}{19}$$

Cancel 27 and one $(1-r)$ factor (since $r \neq 1$ for sum to exist):

$$\frac{(1-r)^2}{1+r+r^2} = \frac{1}{19}$$
$$19(1-r)^2 = 1(1+r+r^2)$$
$$19(1-2r+r^2) = 1+r+r^2$$
$$19 - 38r + 19r^2 = 1+r+r^2$$
$$18r^2 - 39r + 18 = 0$$

Divide by 3: $6r^2 - 13r + 6 = 0$. Factor the quadratic:

$$6r^2 - 9r - 4r + 6 = 0$$
$$3r(2r-3) - 2(2r-3) = 0$$
$$(3r-2)(2r-3) = 0$$

The solutions are $r = 2/3$ or $r = 3/2$. For an infinite G.P. to have a finite sum, we must have $|r| < 1$. Therefore, $r = 2/3$.

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7. Let $a_1, a_2, a_3, \ldots$ be a G.P. of increasing positive terms. If $a_1 a_5 = 81$ and $a_2 + a_4 = 30$, then a_6 is equal to:

(A) 162   (B) 243   (C) 81   (D) 729

Answer: (B)

Solution: Let the G.P. have first term a and common ratio r. We know that for a G.P., $a_1 a_5 = a_3^2$. Given $a_1 a_5 = 81$, so $a_3^2 = 81 \implies a_3 = 9$ (since terms are positive). The second condition is $a_2 + a_4 = 30$. We can write $a_2 = a_3/r$ and $a_4 = a_3 r$.

$$\frac{a_3}{r} + a_3 r = 30$$
$$a_3 \left(\frac{1}{r} + r \right) = 30$$
$$9 \left(\frac{1 + r^2}{r} \right) = 30$$
$$3 \left(\frac{1 + r^2}{r} \right) = 10 \quad \text{(Dividing both sides by 3)}$$
$$3(1 + r^2) = 10r$$
$$3r^2 - 10r + 3 = 0$$

This is a simple quadratic in r. Factorizing it:

$$3r^2 - 9r - r + 3 = 0$$
$$3r(r - 3) - 1(r - 3) = 0$$
$$(3r - 1)(r - 3) = 0$$

This gives two possible values for r: $r = \frac{1}{3}$ or $r = 3$. Since the G.P. is "increasing", we must have $r > 1$. Therefore, $r = 3$. We need to find a_6.

$$a_6 = a_3 \cdot r^3 = 9 \cdot (3)^3$$
$$a_6 = 9 \cdot 27 = 243$$

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8. The geometric mean of $1, 2, 2^2, \ldots, 2^n$ is

(A) $2^{\frac{n}{2}}$
(B) $2^{\frac{(n+1)}{2}}$
(C) $2^{\frac{n(n+1)}{2}}$
(D) $2^{\frac{(n-1)}{2}}$

Answer: (A)

Solution: The sequence is $2^0, 2^1, 2^2, \ldots, 2^n$. First, count the number of terms. This is $n - 0 + 1 = n + 1$ terms. The Geometric Mean (G.M.) is the $(n+1)^{th}$ root of the product of all terms. Product $P = (2^0) \cdot (2^1) \cdot (2^2) \cdots \cdots (2^n)$. When multiplying, we add the exponents:

$$P = 2^{(0 + 1 + 2 + \cdots + n)}$$

The exponent is the sum of the first n integers, which is $\frac{n(n+1)}{2}$.

$$P = 2^{\frac{n(n+1)}{2}}$$

Now, we find the G.M.:

$$\text{G.M.} = (P)^{\frac{1}{n+1}} = \left(2^{\frac{n(n+1)}{2}} \right)^{\frac{1}{n+1}}$$

We multiply the exponents:

$$\text{G.M.} = 2^{\frac{n(n+1)}{2} \cdot \frac{1}{n+1}} = 2^{\frac{n}{2}}$$

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9. In a increasing G.P., $t_2 + t_5 = 216$ and $t_4 : t_6 = 1 : 4$ and all terms are integers, then its first term is

(A) 16
(B) 14
(C) 12
(D) None of these

Answer: (C)
Solution: Let the first term be a and the common ratio be r. From the second condition:

$$\frac{t_4}{t_6} = \frac{ar^3}{ar^5} = \frac{1}{r^2}$$

$$\frac{1}{r^2} = \frac{1}{4} \implies r^2 = 4 \implies r = \pm 2$$

From the first condition:

$$t_2 + t_5 = ar + ar^4 = 216$$
$$ar(1 + r^3) = 216$$

We test the two possible values for r. Case 1: $r = 2$

$$a(2)(1 + 2^3) = 216$$
$$2a(1 + 8) = 216$$
$$2a(9) = 216$$
$$18a = 216$$
$$a = \frac{216}{18} = 12$$

In this case, $a = 12$ and $r = 2$. The terms are $12, 24, 48, \ldots$, which are all integers. This is a valid solution. Case 2: $r = -2$

$$a(-2)(1 + (-2)^3) = 216$$
$$-2a(1 - 8) = 216$$
$$-2a(-7) = 216$$
$$14a = 216$$
$$a = \frac{216}{14} = \frac{108}{7}$$

This is not an integer. If the first term a is not an integer, not all terms will be integers. This case is not valid. Therefore, the first term a must be 12.

10. If five G.M.'s are inserted between 486 and $\frac{2}{3}$, then fourth G.M. will be
    (A) 4   (B) 6   (C) 12   (D) 18

Answer: (B)
Solution: Let $a = 486$ and $b = \frac{2}{3}$. We insert 5 G.M.s ($G_1, \ldots, G_5$) between a and b. The new sequence $a, G_1, G_2, G_3, G_4, G_5, b$ is a G.P. with $n = 5 + 2 = 7$ terms. Here, $a_1 = a = 486$ and $a_7 = b = \frac{2}{3}$. The formula for the nth term is $a_n = a_1 r^{n-1}$.

$$a_7 = a_1 r^{7-1} = a_1 r^6$$
$$\frac{2}{3} = 486 \cdot r^6$$
$$r^6 = \frac{2}{3 \times 486} = \frac{2}{1458} = \frac{1}{729}$$

Since $729 = 3^6$, we have $r^6 = \frac{1}{3^6}$. So, $r = \frac{1}{3}$ (assuming positive G.M.s, as implied by options). The kth G.M. is $G_k = ar^k$. We need to find the 4th G.M., G_4.

$$G_4 = ar^4 = 486 \left(\frac{1}{3}\right)^4 = \frac{486}{3^4} = \frac{486}{81}$$

Since $81 \times 6 = (80 + 1) \times 6 = 480 + 6 = 486$.

$$G_4 = 6$$

11. An A.P. consists of 23 terms. If the sum of the three terms in the middle is 141 and the sum of the last three terms is 261, then the first term is

(A) 6   (B) 5   (C) 4   (D) 3

> **Answer: (D)**
>
> **Solution:** Given $n = 23$ terms. The middle term is $a_{\left(\frac{n+1}{2}\right)} = a_{\left(\frac{23+1}{2}\right)} = a_{12}$. The three terms in the middle are a_{11}, a_{12}, a_{13}. Their sum is:
>
> $$a_{11} + a_{12} + a_{13} = (a + 10d) + (a + 11d) + (a + 12d) = 3a + 33d = 141$$
>
> Dividing by 3, we get $a + 11d = 47$. (This is a_{12}) $\cdots$ (1) The last three terms are a_{21}, a_{22}, a_{23}. Their sum is:
>
> $$a_{21} + a_{22} + a_{23} = (a + 20d) + (a + 21d) + (a + 22d) = 3a + 63d = 261$$
>
> Dividing by 3, we get $a + 21d = 87$. (This is a_{22}) $\cdots$ (2) We now have a system of two equations. Subtract (1) from (2):
>
> $$(a + 21d) - (a + 11d) = 87 - 47$$
> $$10d = 40$$
> $$d = 4$$
>
> Substitute $d = 4$ into (1) to find a:
>
> $$a + 11(4) = 47$$
> $$a + 44 = 47$$
> $$a = 3$$
>
> The first term is 3.

12. A man saves Rs. 200 in each of the first three months of his service. In each of the subsequent months his saving increases by Rs. 40 more than the saving of immediately previous month. His total saving from the start of service will be Rs. 11040 after

(A) 18 months   (B) 19 months   (C) 20 months   (D) 21 months

> **Answer: (D)**
>
> **Solution:** Saving in first 3 months $= 200 + 200 + 200 =$ Rs. 600. Total saving needed $=$ Rs. 11040. Remaining saving needed $= 11040 - 600 =$ Rs. 10440. The savings from the 4th month onwards form an A.P. Saving in 4th month $= 200 + 40 = 240$. Saving in 5th month $= 240 + 40 = 280$. This is an A.P. with first term $a = 240$, common difference $d = 40$. Let this A.P. continue for k months. The sum S_k must be 10440.
>
> $$S_k = \frac{k}{2}[2a + (k-1)d] = 10440$$
> $$\frac{k}{2}[2(240) + (k-1)40] = 10440$$
> $$\frac{k}{2}[480 + 40k - 40] = 10440$$
> $$\frac{k}{2}[440 + 40k] = 10440$$
> $$k(220 + 20k) = 10440$$
>
> Divide by 20:
>
> $$k(11 + k) = \frac{10440}{20} = 522$$
> $$k^2 + 11k - 522 = 0$$
>
> We factor the quadratic. We need two numbers with product -522 and sum 11. We can test 18 and 29: $29 \times 18 = 522$.
>
> $$k^2 + 29k - 18k - 522 = 0$$

$$k(k+29) - 18(k+29) = 0$$
$$(k-18)(k+29) = 0$$

Since k (number of months) must be positive, $k = 18$. Total time = (first 3 months) + (k subsequent months) $= 3 + 18 = 21$ months.

13. If $log_3 2, log_3(2^x - 5)$ and $log_3(2^x - \frac{7}{2})$ are in A.P., then x is equal to
(A) 1   (B) 3/2   (C) 2   (D) 3

Answer: (D)
Solution: If a, b, c are in A.P., then $2b = a + c$.

$$2\log_3(2^x - 5) = \log_3(2) + \log_3(2^x - 7/2)$$

Using logarithm properties $\log(b^k) = k \log b$ and $\log a + \log b = \log ab$:

$$\log_3((2^x - 5)^2) = \log_3(2 \cdot (2^x - 7/2))$$

Since $\log_3(A) = \log_3(B) \implies A = B$:

$$(2^x - 5)^2 = 2(2^x - 7/2)$$
$$(2^x - 5)^2 = 2 \cdot 2^x - 7$$

Let $y = 2^x$.

$$(y - 5)^2 = 2y - 7$$
$$y^2 - 10y + 25 = 2y - 7$$
$$y^2 - 12y + 32 = 0$$

Factor the quadratic:

$$(y - 4)(y - 8) = 0$$

This gives two possible solutions for y: $y = 4$ or $y = 8$. Case 1: $y = 4$

$$2^x = 4 \implies x = 2$$

We must check if this is valid in the original logarithms. $\log_3(2^x - 5) = \log_3(4 - 5) = \log_3(-1)$. This is undefined. So $x = 2$ is not a solution.
Case 2: $y = 8$

$$2^x = 8 \implies x = 3$$

Check validity: $\log_3(2^x - 5) = \log_3(8 - 5) = \log_3(3)$. This is defined. $\log_3(2^x - 7/2) = \log_3(8 - 3.5) = \log_3(4.5)$. This is also defined. Therefore, $x = 3$ is the only valid solution.