HI EVERYONE,

THE REAL LEARNING IN MATHEMATICS HAPPENS WHEN YOU ACTIVELY ENGAGE WITH A PROBLEM, EXPLORE DIFFERENT METHODS, AND WORK THROUGH CHALLENGES. THEREFORE, WE STRONGLY ENCOURAGE YOU TO USE THIS SOLUTION KEY RESPONSIBLY.

PLEASE ATTEMPT ALL THE PROBLEMS ON YOUR OWN FIRST, GIVING THEM YOUR BEST AND MOST HONEST EFFORT. THESE SOLUTIONS ARE TO HELP YOU GET UNSTUCK ON A PROBLEM AFTER YOU HAVE ALREADY TRIED YOUR BEST.

YOUR EFFORT AND DEDICATION ARE THE TRUE KEYS TO SUCCESS.

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Syllabus: Sequences & Series

Sub: Mathematics

CT-09 JEE Main Star - Solution

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1. If $\frac{3+5+7+\ldots \text{ to } n \text{ terms}}{5+8+11+\ldots \text{ to } 10 \text{ terms}} = 7$, then n is equal to

(A) 35 (B) 36 (C) 37 (D) 38

Answer: (A)
Solution: The numerator is the sum of an A.P. with $a_1 = 3$, $d_1 = 2$, and $n_1 = n$ terms.

$$\text{Sum}_1 = \frac{n}{2}[2(3) + (n-1)2] = \frac{n}{2}[6 + 2n - 2] = \frac{n}{2}[2n+4] = n(n+2)$$

The denominator is the sum of an A.P. with $a_2 = 5$, $d_2 = 3$, and $n_2 = 10$ terms.

$$\text{Sum}_2 = \frac{10}{2}[2(5) + (10-1)3] = 5[10 + 9(3)] = 5[10+27] = 5(37) = 185$$

We are given the ratio of the sums is 7.

$$\frac{\text{Sum}_1}{\text{Sum}_2} = \frac{n(n+2)}{185} = 7$$

Now, we solve for n:

$$n(n+2) = 7 \times 185$$
$$n^2 + 2n = 1295$$
$$n^2 + 2n - 1295 = 0$$

We factor the quadratic. We look for two numbers with a product of -1295 and a difference of 2. These are 37 and -35.

$$(n+37)(n-35) = 0$$

Since n represents the number of terms, it must be positive. Therefore, $n = 35$.

2. If n A.M.'s are inserted between 3 and 17 such that the ratio of the last mean to the first mean is 3: 1, then the value of n is

(A) 4 (B) 6 (C) 8 (D) 9

Answer: (B)
Solution: When n A.M.'s are inserted between $a = 3$ and $b = 17$, the common difference d is:

$$d = \frac{b-a}{n+1} = \frac{17-3}{n+1} = \frac{14}{n+1}$$

The first mean, A_1, is $a + d$.

$$A_1 = 3 + d = 3 + \frac{14}{n+1}$$

The last mean, A_n, is $a + nd$.

$$A_n = 3 + nd = 3 + n\left(\frac{14}{n+1}\right)$$

We are given that $\frac{A_n}{A_1} = \frac{3}{1}$.

$$\frac{3 + \frac{14n}{n+1}}{3 + \frac{14}{n+1}} = 3$$

Multiply the numerator and denominator of the left side by $(n + 1)$:

$$\frac{3(n+1) + 14n}{3(n+1) + 14} = 3$$
$$\frac{3n + 3 + 14n}{3n + 3 + 14} = 3$$
$$\frac{17n + 3}{3n + 17} = 3$$

Now, solve for n:

$$17n + 3 = 3(3n + 17)$$
$$17n + 3 = 9n + 51$$
$$8n = 48$$
$$n = 6$$

3. If S_n denotes the sum to n terms of an A.P., then $S_{n+3} - 3S_{n+2} + 3S_{n+1} - S_n$ is equal to

(A) −d (B) −a (C) 0 (D) 2a

Answer: (C)

Solution: Let T_n be the n^{th} term of the A.P. We know that $T_n = S_n - S_{n-1}$. Let's analyze the given expression by grouping the terms:

$$\text{Expr} = (S_{n+3} - 3S_{n+2} + 3S_{n+1} - S_n)$$

We can rewrite this as:

$$\text{Expr} = (S_{n+3} - S_{n+2}) - 2(S_{n+2} - S_{n+1}) + (S_{n+1} - S_n)$$

Now, substitute the definition of the n^{th} term:

$$S_{n+3} - S_{n+2} = T_{n+3}$$
$$S_{n+2} - S_{n+1} = T_{n+2}$$
$$S_{n+1} - S_n = T_{n+1}$$

Substitute these back into the expression:

$$\text{Expr} = T_{n+3} - 2T_{n+2} + T_{n+1}$$

Now, express the T_n terms using the first term a and common difference d:

$$T_{n+3} = a + (n + 3 - 1)d = a + (n + 2)d$$
$$T_{n+2} = a + (n + 2 - 1)d = a + (n + 1)d$$
$$T_{n+1} = a + (n + 1 - 1)d = a + nd$$

Substitute these into our expression for the T terms:

$$\begin{aligned}\text{Expr} &= [a + (n + 2)d] - 2[a + (n + 1)d] + [a + nd] \\ &= (a - 2a + a) + d \cdot [(n + 2) - 2(n + 1) + n] \\ &= 0 + d \cdot [n + 2 - 2n - 2 + n] \\ &= 0\end{aligned}$$

4. If a_n be the n^{th} term of an AP and if $a_7 = 15$, then the value of the common difference that would make $a_2a_7a_{12}$ greatest is

Answer: 0
Solution: We are given $a_7 = 15$. Let the common difference be d. We can express a_2 and a_{12} in terms of a_7:

$$a_2 = a_7 - 5d = 15 - 5d$$
$$a_{12} = a_7 + 5d = 15 + 5d$$

We want to find the value of d that maximizes the product $P = a_2a_7a_{12}$.

$$P = (15 - 5d)(15)(15 + 5d)$$
$$P = 15 \cdot (15 - 5d)(15 + 5d)$$

Using the difference of squares formula $(x - y)(x + y) = x^2 - y^2$:

$$P = 15 \cdot [(15)^2 - (5d)^2]$$
$$P = 15 \cdot [225 - 25d^2]$$
$$P = 375(9 - d^2)$$

To maximize P, we must maximize $(9 - d^2)$. Since $d^2 \geq 0$, the maximum value occurs when d^2 is minimum. Thus, $d = 0$.

5. Let $a_1, a_2, a_3, \ldots$ be in harmonic progression with $a_1 = 5$ and $a_{20} = 25$. The least positive integer n for which $a_n < 0$ is

(A) 22 (B) 23 (C) 24 (D) 25

Answer: (D)
Solution: If a_n is in H.P., then $b_n = 1/a_n$ is in A.P. Let b_n be the A.P. with first term b_1 and common difference d.

$$b_1 = \frac{1}{a_1} = \frac{1}{5}, \quad b_{20} = \frac{1}{a_{20}} = \frac{1}{25}$$

Using the A.P. formula $b_{20} = b_1 + (20 - 1)d$:

$$\frac{1}{25} = \frac{1}{5} + 19d$$
$$19d = \frac{1}{25} - \frac{5}{25} = -\frac{4}{25}$$
$$d = -\frac{4}{475}$$

We want $a_n < 0$, which implies $b_n < 0$:

$$b_1 + (n - 1)d < 0$$
$$\frac{1}{5} + (n - 1)\left(-\frac{4}{475}\right) < 0$$
$$\frac{1}{5} < (n - 1)\left(\frac{4}{475}\right)$$
$$\frac{475}{20} < n - 1$$
$$23.75 < n - 1$$
$$24.75 < n$$

The least integer n greater than 24.75 is 25.

6. If 1, $log_9(3^{1-x}+2)$ and $log_3(4\cdot 3^x - 1)$ are in A.P., then x is equal to

(A) $log_3 4$ (B) $1 - log_3 4$ (C) $log_3 0.25$ (D) $log_4 3$

Answer: (B)
Solution: If a, b, c are in A.P., then $2b = a + c$.

$$2\log_9(3^{1-x}+2) = 1 + \log_3(4\cdot 3^x - 1)$$

Since $\log_9(y) = \frac{1}{2}\log_3(y)$, the LHS becomes $\log_3(3^{1-x}+2)$.

$$\log_3(3^{1-x}+2) = \log_3(3) + \log_3(4\cdot 3^x - 1)$$
$$\log_3(3^{1-x}+2) = \log_3[3(4\cdot 3^x - 1)]$$

Equate arguments and let $y = 3^x$:

$$\frac{3}{y} + 2 = 12y - 3$$
$$3 + 2y = 12y^2 - 3y$$
$$12y^2 - 5y - 3 = 0$$
$$(3y+1)(4y-3) = 0$$

Since $y = 3^x$ must be positive, $y = 3/4$.

$$3^x = \frac{3}{4} \implies x = \log_3(3) - \log_3(4) = 1 - \log_3(4)$$

7. Three numbers whose product is 512 are in G.P. If 8 is added to the first and 6 to the second, the numbers will be in A.P. The numbers are

(A) 2, 8, 32 (B) 8, 8, 8 (C) 4, 8, 16 (D) 2, 8, 14

Answer: (C)
Solution: Let the numbers in G.P. be $\frac{a}{r}, a, ar$. Product is $a^3 = 512 \implies a = 8$. Terms are $8/r, 8, 8r$. New terms for A.P.: $\frac{8}{r} + 8$, 14, $8r$. Condition for A.P. ($2b = a + c$):

$$2(14) = \left(\frac{8}{r} + 8\right) + 8r$$
$$28 = \frac{8}{r} + 8 + 8r$$
$$20 = 8\left(r + \frac{1}{r}\right)$$
$$\frac{5}{2} = \frac{r^2+1}{r}$$
$$2r^2 - 5r + 2 = 0$$

$(2r-1)(r-2) = 0$, so $r = 2$ or $r = 1/2$. Both give the set $\{4, 8, 16\}$.

8. Let a_n be the n^{th} term of the G.P. of positive numbers. Let $\sum_{n=1}^{100} a_{2n} = \alpha$ and $\sum_{n=1}^{100} a_{2n-1} = \beta$ such that $\alpha \neq \beta$, then the common ratio is

(A) $\frac{\alpha}{\beta}$ (B) $\frac{\beta}{\alpha}$ (C) $\sqrt{\frac{\alpha}{\beta}}$ (D) $\sqrt{\frac{\beta}{\alpha}}$

Answer: (A)

Solution: Let the G.P. be $a, ar, ar^2, \ldots$. α is the sum of even-indexed terms $(ar, ar^3, \ldots)$:

$$\alpha = ar + ar^3 + \cdots + ar^{199} = r(a + ar^2 + \cdots + ar^{198})$$

β is the sum of odd-indexed terms $(a, ar^2, \ldots)$:

$$\beta = a + ar^2 + \cdots + ar^{198}$$

From the equation for α, we can see $\alpha = r \cdot \beta$.

$$r = \frac{\alpha}{\beta}$$

9. If the sixth term of a GP be 2, then the product of first eleven terms is
(A) 1024 (B) 2047 (C) 2048 (D) 1023

Answer: (C)
Solution: Let the G.P. be $a, ar, \ldots$. Given $T_6 = ar^5 = 2$. Product of first 11 terms:

$$P_{11} = (a)(ar)(ar^2)\ldots(ar^{10})$$
$$P_{11} = a^{11} \cdot r^{0+1+\cdots+10}$$
$$P_{11} = a^{11}r^{55}$$

We can factor this as:

$$P_{11} = (ar^5)^{11}$$

Substitute $ar^5 = 2$:

$$P_{11} = 2^{11} = 2048$$

10. If the ratio of the sum of n terms of two A.P.'s are $3n + 8$ and $7n + 15$, then the ratio of their 12^{th} term will be
(A) $\frac{4}{9}$ (B) $\frac{7}{16}$ (C) $\frac{3}{7}$ (D) $\frac{8}{15}$

Answer: (B)
Solution: Let the sums be S_n and S'_n.

$$\frac{S_n}{S'_n} = \frac{\frac{n}{2}[2a + (n-1)d]}{\frac{n}{2}[2a' + (n-1)d']} = \frac{2a + (n-1)d}{2a' + (n-1)d'} = \frac{3n+8}{7n+15}$$

To find the ratio of the 12^{th} terms $\frac{a_{12}}{a'_{12}} = \frac{a+11d}{a'+11d'}$, we divide the LHS numerator and denominator by 2:

$$\frac{a + \frac{n-1}{2}d}{a' + \frac{n-1}{2}d'}$$

We set $\frac{n-1}{2} = 11 \implies n = 23$. Substitute $n = 23$ into the RHS:

$$\text{Ratio} = \frac{3(23)+8}{7(23)+15} = \frac{69+8}{161+15} = \frac{77}{176} = \frac{7}{16}$$

11. The interior angles of a polygon are in A.P. If the smallest angle be 120° and the common difference be 5°, then the number of sides is

(A) 8 (B) 10 (C) 9 (D) 6

Answer: (C)
Solution: Let n be the number of sides. Sum of interior angles is $(n-2)180$. Using A.P. sum formula:

$$\begin{aligned}\frac{n}{2}[2(120)+(n-1)5] &= 180(n-2)\\ n(240+5n-5) &= 360n-720\\ 5n^2+235n &= 360n-720\\ 5n^2-125n+720 &= 0\\ n^2-25n+144 &= 0\\ (n-16)(n-9) &= 0\end{aligned}$$

If $n=16$, largest angle $a_{16}=120+15(5)=195^\circ>180^\circ$, which is invalid. Thus, $n=9$.

12. A man saves Rs. 200 in each of the first three months of his service. In each of the subsequent months his saving increases by Rs. 40 more than the saving of immediately previous month. His total saving from the start of service will be Rs. 11040 after

(A) 18 months (B) 19 months (C) 20 months (D) 21 months

Answer: (D)
Solution: First 3 months sum $=600$. Remaining sum needed $=11040-600=10440$. From 4th month, it is an A.P. with $a=240, d=40$. Let this last for k months.

$$\begin{aligned}\frac{k}{2}[2(240)+(k-1)40] &= 10440\\ k[240+20(k-1)] &= 10440\\ k(220+20k) &= 10440\\ 20k(11+k) &= 10440\\ k^2+11k-522 &= 0\\ (k+29)(k-18) &= 0\end{aligned}$$

So, $k=18$ months. Total time $=3+18=21$ months.

13. Let $S_1, S_2, \ldots, S_{101}$ be consecutive terms of an A.P. If $\frac{1}{S_1S_2}+\frac{1}{S_2S_3}+\cdots+\frac{1}{S_{100}S_{101}}=\frac{1}{6}$ and $S_1+S_{101}=50$, then $|S_1-S_{101}|$ is equal to

(A) 10 (B) 20 (C) 30 (D) 40

Answer: (A)
Solution: Let d be the common difference. The general term is:

$$\frac{1}{S_kS_{k+1}}=\frac{1}{d}\left(\frac{S_{k+1}-S_k}{S_kS_{k+1}}\right)=\frac{1}{d}\left(\frac{1}{S_k}-\frac{1}{S_{k+1}}\right)$$

The sum telescopes:

$$\begin{aligned}\text{Sum} &= \frac{1}{d}\left[\left(\frac{1}{S_1}-\frac{1}{S_2}\right)+\cdots+\left(\frac{1}{S_{100}}-\frac{1}{S_{101}}\right)\right]\\ &= \frac{1}{d}\left(\frac{1}{S_1}-\frac{1}{S_{101}}\right)=\frac{1}{d}\left(\frac{S_{101}-S_1}{S_1S_{101}}\right)\end{aligned}$$

Since $S_{101} - S_1 = 100d$, the sum simplifies to $\frac{100}{S_1 S_{101}}$.

$$\frac{100}{S_1 S_{101}} = \frac{1}{6} \implies S_1 S_{101} = 600$$

Given $S_1 + S_{101} = 50$. We need $|S_1 - S_{101}|$.

$$\begin{aligned}(S_1 - S_{101})^2 &= (S_1 + S_{101})^2 - 4S_1 S_{101} \\ &= (50)^2 - 4(600) = 2500 - 2400 = 100 \\ |S_1 - S_{101}| &= \sqrt{100} = 10\end{aligned}$$