Hi everyone,

The real learning in mathematics happens when you actively engage with a problem, explore different methods, and work through challenges. Therefore, we strongly encourage you to use this solution key responsibly.

Please attempt all the problems on your own first, giving them your best and most honest effort. These solutions are to help you get unstuck on a problem after you have already tried your best.

Your effort and dedication are the true keys to success.

Exam Date: 16th November 2025

Syllabus: Functions & ITF

Sub: Mathematics **CT-15 JEE Main Regular - Solution** Prof. Chetan Sir

---

1. If the domain of the function $f(x) = \sin^{-1}\left(\frac{x-1}{2x+3}\right)$ is $R - (\alpha, \beta)$ then $12\alpha\beta$ is equal to:

(A) 32 (B) 40 (C) 24 (D) 36

Answer: (A)
Solution: For $\sin^{-1}(A)$, we must have $-1 \le A \le 1$.

$$-1 \le \frac{x-1}{2x+3} \le 1 \tag{1}$$

Case 1: $\frac{x-1}{2x+3} \le 1$

$$\begin{aligned}\frac{x-1}{2x+3} - 1 &\le 0\\ \frac{x-1-2x-3}{2x+3} &\le 0\\ \frac{-(x+4)}{2x+3} &\le 0\\ \frac{x+4}{2x+3} &\ge 0 \end{aligned}\tag{2}$$

Solution: $(-\infty, -4] \cup (-3/2, \infty)$.
Case 2: $\frac{x-1}{2x+3} \ge -1$

$$\begin{aligned}\frac{x-1}{2x+3} + 1 &\ge 0\\ \frac{x-1+2x+3}{2x+3} &\ge 0\\ \frac{3x+2}{2x+3} &\ge 0 \end{aligned}\tag{3}$$

Solution: $(-\infty, -3/2) \cup [-2/3, \infty)$.
Intersection of Case 1 and Case 2: $(-\infty, -4] \cup [-2/3, \infty)$. The domain is $\mathbb{R} - (-4, -2/3)$. Comparing with $\mathbb{R} - (\alpha, \beta)$, we get $\alpha = -4$ and $\beta = -2/3$.

$$12\alpha\beta = 12(-4)(-2/3) = 12(8/3) = 32 \tag{4}$$

2. Let $f : R \to R$ be defined by $f(x) = \frac{x}{1+x^2}$, $x \in R$. Then the range of f is:

(A) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (B) $R - [-1, 1]$
(C) $R - \left[-\frac{1}{2}, \frac{1}{2}\right]$ (D) $(-1, 1) - \{0\}$

Answer: (A)

Solution: Let $y = \frac{x}{1+x^2}$.

$$\begin{aligned} y(1+x^2) &= x \\ yx^2 - x + y &= 0 \end{aligned} \tag{5}$$

For x to be real, the discriminant $D \geq 0$.

$$\begin{aligned} (-1)^2 - 4(y)(y) &\geq 0 \\ 1 - 4y^2 &\geq 0 \\ 4y^2 &\leq 1 \\ y^2 &\leq \frac{1}{4} \\ -\frac{1}{2} &\leq y \leq \frac{1}{2} \end{aligned} \tag{6}$$

Range is $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

3. If $f(x) = \frac{4x+3}{6x-4}$, $x \neq \frac{2}{3}$ and $(fof)(x) = g(x)$, where $g : R - \{\frac{2}{3}\} \to R - \{\frac{2}{3}\}$, then (gogog) (4) is equal to:

(A) $-\frac{19}{20}$ (B) $\frac{19}{20}$ (C) -4 (D) 4

Answer: (D)
Solution: First, compute $(fof)(x)$:

$$\begin{aligned} f(f(x)) &= \frac{4\left(\frac{4x+3}{6x-4}\right) + 3}{6\left(\frac{4x+3}{6x-4}\right) - 4} \\ &= \frac{4(4x+3) + 3(6x-4)}{6(4x+3) - 4(6x-4)} \\ &= \frac{16x + 12 + 18x - 12}{24x + 18 - 24x + 16} \\ &= \frac{34x}{34} = x \end{aligned} \tag{7}$$

So, $g(x) = x$ is the identity function. Therefore, $(gogog)(4) = g(g(g(4))) = 4$.

4. If $f(x) = \log_e\left(\frac{1-x}{1+x}\right)$, $|x| < 1$, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to:

(A) $(f(x))^2$ (B) $2f(x^2)$ (C) $-2f(x)$ (D) $2f(x)$

Answer: (D)
Solution: Substitute argument into function:

$$\begin{aligned} f\left(\frac{2x}{1+x^2}\right) &= \log_e\left(\frac{1 - \frac{2x}{1+x^2}}{1 + \frac{2x}{1+x^2}}\right) \\ &= \log_e\left(\frac{1 + x^2 - 2x}{1 + x^2 + 2x}\right) \\ &= \log_e\left(\frac{(1-x)^2}{(1+x)^2}\right) \\ &= \log_e\left(\frac{1-x}{1+x}\right)^2 \end{aligned}$$

$$= 2\log_e\left(\frac{1-x}{1+x}\right) = 2f(x) \qquad (8)$$

5. The function $f : N - \{1\} \to N$; defined by $f(n) =$ the highest prime factor of n, is:

(A) both one-one and onto
(B) one-one only
(C) onto only
(D) neither one-one nor onto

Answer: (D)
Solution: One-one check: $f(6) = 3$ (factors of 6 are 2, 3) and $f(9) = 3$ (factors of 9 are 3). Since $6 \neq 9$ but $f(6) = f(9)$, it is not one-one.
Onto check: The codomain is N. Can we find n such that $f(n) = 4$? No, because 4 is not a prime number. The range consists only of prime numbers. Thus, range $\neq$ codomain. Not onto.

6. The function $f : R \to R$, $f(x) = \frac{x^2+2x-15}{x^2-4x+9}$, $x \in \mathbb{R}$ is:

(A) one-one but not onto
(B) both one-one and onto
(C) onto but not one-one
(D) neither one-one nor onto

Answer: (D)
Solution: Onto check: Let $y = \frac{x^2+2x-15}{x^2-4x+9}$. For y to be in the range, x must be real ($D \geq 0$ for the resulting quadratic in x). Solving for range gives $y \in [-2, 1.6]$ approximately. Since the codomain is $\mathbb{R}$ and range is a subset, it is **not onto**.
One-one check: Since the function is continuous and goes up and down within a closed interval range, it takes the same y-value for multiple x-values. It is **not one-one**.

7. Let $f : R \to R$: and $g : R \to R$ be defined as $f(x) = \begin{cases} \log_e x, & x > 0 \\ e^{-x}, & x \leq 0 \end{cases}$ and $g(x) = \begin{cases} x, & x \geq 0 \\ e^x, & x < 0 \end{cases}$. Then, gof: $R \to R$:

(A) one-one but not onto
(B) neither one-one nor onto
(C) onto but not one-one
(D) both one-one and onto

Answer: (B)
Solution: Calculate $g(f(x))$: If $x \leq 0$, $f(x) = e^{-x} \geq 1$. Since result > 0, $g(e^{-x}) = e^{-x}$. If $0 < x < 1$, $f(x) = \ln x (< 0)$. Input to g is negative. $g(f(x)) = e^{\ln x} = x$. If $x \geq 1$, $f(x) = \ln x (\geq 0)$. Input to g is positive. $g(f(x)) = \ln x$.
One-one: $g(f(x))$ takes values in $[0, \infty)$. $g(f(-1)) = e^1 = e$. $g(f(e)) = \ln e = 1$. $g(f(x)) = 2$ has solutions in both $x \leq 0$ branch and $x \geq 1$ branch. Not one-one.
Onto: Range is $[0, \infty)$. Codomain is $\mathbb{R}$. Not onto.

8. The number of distinct real roots of the equation $|x||x + 2| - 5|x + 1| - 1 = 0$ is:

(A) 3
(B) 2
(C) 4
(D) 1

Answer: (A)
Solution: We analyze cases based on critical points $x = 0, -2, -1$.
Case 1 ($x \geq 0$):

$$x(x + 2) - 5(x + 1) - 1 = 0$$

$$x^2 - 3x - 6 = 0 \tag{9}$$

Positive root $x = \frac{3+\sqrt{33}}{2}$. (1 sol)
Case 2 $(-1 \le x < 0)$:

$$\begin{aligned} -x(x+2) - 5(x+1) - 1 &= 0 \\ x^2 + 7x + 6 &= 0 \end{aligned} \tag{10}$$

Root $x = -1$ is valid. (1 sol)
Case 3 $(-2 \le x < -1)$:

$$\begin{aligned} -x(x+2) - 5(-x-1) - 1 &= 0 \\ x^2 - 3x - 4 &= 0 \end{aligned} \tag{11}$$

Roots $4, -1$. Neither in interval $(-2, -1)$. (0 sol)
Case 4 $(x < -2)$:

$$\begin{aligned} -x(-x-2) - 5(-x-1) - 1 &= 0 \\ x^2 + 7x + 4 &= 0 \end{aligned} \tag{12}$$

Root $\frac{-7-\sqrt{33}}{2} \approx -6.37$ is valid. (1 sol)
Total = 3 distinct roots.

9. Let f be a function such that $f(x) + 3f\left(\frac{24}{x}\right) = 4x$, $x \neq 0$. Then $f(3) + f(8)$ is equal to:
(A) 11 (B) 10 (C) 12 (D) 13

Answer: (A)
Solution: Given:

$$f(x) + 3f(24/x) = 4x \tag{13}$$

Replace x with $24/x$:

$$f(24/x) + 3f(x) = 4(24/x) = 96/x \tag{14}$$

Multiply (9) by 3:

$$3f(x) + 9f(24/x) = 12x \tag{15}$$

Subtract (10) from (11):

$$\begin{aligned} 8f(24/x) &= 12x - 96/x \\ f(24/x) &= 1.5x - 12/x \end{aligned} \tag{16}$$

Substitute back into (9):

$$\begin{aligned} f(x) + 3(1.5x - 12/x) &= 4x \\ f(x) &= 36/x - 0.5x \end{aligned} \tag{17}$$

$$\begin{aligned} f(3) &= 12 - 1.5 = 10.5 \\ f(8) &= 36/8 - 4 = 4.5 - 4 = 0.5 \\ f(3) + f(8) &= 11 \end{aligned} \tag{18}$$

10. The inverse function of $f(x) = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}$, $x \in (-1, 1)$ is:

(A) $\frac{1}{4}\log_e\left(\frac{1+x}{1-x}\right)$ (B) $\frac{1}{4}\log_e\left(\frac{1-x}{1+x}\right)$

(C) $\frac{1}{4}(\log_e)\log_e\left(\frac{1-x}{1+x}\right)$ (D) $\frac{1}{4}\log_8\left(\frac{1+x}{1-x}\right)$

Answer: (D)
Solution: Let $y = \frac{a-1/a}{a+1/a} = \frac{a^2-1}{a^2+1}$ where $a = 8^{2x}$. Solving for a^2:

$$y(a^2+1) = a^2 - 1$$
$$a^2(1-y) = 1+y$$
$$a^2 = \frac{1+y}{1-y} \tag{19}$$

$$(8^{2x})^2 = 8^{4x} = \frac{1+y}{1-y} \tag{20}$$

Take $\log_8$:

$$4x = \log_8\left(\frac{1+y}{1-y}\right)$$
$$x = \frac{1}{4}\log_8\left(\frac{1+y}{1-y}\right) \tag{21}$$

11. Consider the function $f : \left[\frac{1}{2}, 1\right] \to R$ defined by $f(x) = 4\sqrt{2}x^3 - 3\sqrt{2}x - 1$. Consider the following statements:
(I) The curve $y = f(x)$ intersects the x-axis exactly at one point
(II) The curve $y = f(x)$ intersects the x-axis at $x = \cos\left(\frac{\pi}{12}\right)$
Then which of the following is true?
(A) Only (II) is correct (B) Both (I) and (II) are incorrect
(C) Only (I) is correct (D) Both (I) and (II) are correct

Answer: (D)
Solution: $f(x) = \sqrt{2}(4x^3 - 3x) - 1$. Let $x = \cos\theta$. For $x \in [0.5, 1]$, $\theta \in [0, \pi/3]$.

$$f(x) = \sqrt{2}\cos(3\theta) - 1 = 0$$
$$\cos(3\theta) = \frac{1}{\sqrt{2}}$$
$$3\theta = \frac{\pi}{4}$$
$$\theta = \frac{\pi}{12} \tag{22}$$

This is the only solution in range $[0, \pi]$. Intersection point is $x = \cos\left(\frac{\pi}{12}\right)$. (I) is correct (one point), (II) is correct.

12. If the inverse trigonometric functions take principal values, then $\cos^{-1}\left(\frac{3}{10}\cos\left(\tan^{-1}\left(\frac{4}{3}\right)\right) + \frac{2}{5}\sin\left(\tan^{-1}\left(\frac{4}{3}\right)\right)\right)$ is equal to:
(A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

Answer: (C)
Solution: Let $\theta = \tan^{-1}(4/3)$. Then $\sin\theta = 4/5$, $\cos\theta = 3/5$.

$$\text{Expression} = \frac{3}{10}\left(\frac{3}{5}\right) + \frac{2}{5}\left(\frac{4}{5}\right)$$

$$= \frac{9}{50} + \frac{8}{25} = \frac{9+16}{50} = \frac{25}{50} = \frac{1}{2} \tag{23}$$

$$\cos^{-1}(1/2) = \frac{\pi}{3} \tag{24}$$

13. Considering only the principal values of the inverse trigonometric functions, the number of values of x satisfying $\tan^{-1}(x) + \tan^{-1}(2x) = \frac{\pi}{4}$ is:

(A) More than 2 (B) 1 (C) 2 (D) 0

Answer: (B)
Solution:

$$\tan^{-1}\left(\frac{x+2x}{1-2x^2}\right) = \frac{\pi}{4}$$
$$\frac{3x}{1-2x^2} = 1$$
$$2x^2 + 3x - 1 = 0$$
$$x = \frac{-3 \pm \sqrt{9+8}}{4} \tag{25}$$

For positive real values (required for $\pi/4$ sum), $x = \frac{\sqrt{17}-3}{4}$. Only 1 positive solution.

14. Given that the inverse trigonometric function assumes principal values only. Let x, y be any two real numbers in $[-1, 1]$ such that $\cos^{-1} x - \sin^{-1} y = \alpha$, then the minimum value of $x^2 + y^2 + 2xy \sin\alpha$ is equal to:

(A) 0 (B) -1 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

Answer: (A)
Solution: Let $A = \cos^{-1} x$ and $B = \sin^{-1} y$. $A - B = \alpha$. $x = \cos A$, $y = \sin B$. Using expansion: $\cos(A - B) = \cos\alpha$.

$$\cos A \cos B + \sin A \sin B = \cos\alpha$$
$$x\sqrt{1-y^2} + \sqrt{1-x^2}y = \cos\alpha \tag{26}$$

Let $x = 1, y = 1$. $\alpha = 0 - \pi/2 = -\pi/2$. Expr $= 1 + 1 + 2(-1) = 0$. Let $x = 0, y = 0$. $\alpha = \pi/2 - 0 = \pi/2$. Expr $= 0 + 0 + 0 = 0$. Generally, $x^2 + y^2 + 2xy \sin\alpha = \cos^2\alpha$. Minimum of $\cos^2\alpha$ is 0.

15. Considering the principal values of the inverse trigonometric functions, $\sin^{-1}\left(\frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{1-x^2}\right)$, where $-\frac{1}{2} < x < \frac{1}{\sqrt{2}}$, is equal to:

(A) $\frac{\pi}{4} + \sin^{-1} x$ (B) $\frac{\pi}{6} + \sin^{-1} x$
(C) $\frac{-5\pi}{6} - \sin^{-1} x$ (D) $\frac{5\pi}{6} - \sin^{-1} x$

Answer: (B)
Solution: Let $x = \sin\theta$. Since $-\frac{1}{2} < x < \frac{1}{\sqrt{2}}$, we have $-\frac{\pi}{6} < \theta < \frac{\pi}{4}$.

$$\text{Exp} = \sin^{-1}\left(\sin\frac{\pi}{3}\sin\theta + \cos\frac{\pi}{3}\cos\theta\right)$$
$$= \sin^{-1}\left(\cos\left(\theta - \frac{\pi}{3}\right)\right)$$

$$= \sin^{-1}\left(\sin\left(\frac{\pi}{2} - \left(\theta - \frac{\pi}{3}\right)\right)\right)$$

$$= \sin^{-1}\left(\sin\left(\frac{5\pi}{6} - \theta\right)\right) \tag{27}$$

Range of argument $\frac{5\pi}{6} - \theta$: $\left(\frac{7\pi}{12}, \pi\right)$. This is in 2nd quadrant. $\sin^{-1}(\sin A) = \pi - A$ in 2nd quadrant.

$$\text{Value} = \pi - \left(\frac{5\pi}{6} - \theta\right) = \frac{\pi}{6} + \theta = \frac{\pi}{6} + \sin^{-1} x \tag{28}$$

16. Considering the principal values of the inverse trigonometric functions, the sum of all the values of x satisfying $\cos^{-1}(x) - 2\ \sin^{-1}(x) = \cos^{-1}(2x)$ is equal to:

(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

Answer: (A)
Solution: Let $\sin^{-1} x = \theta \implies x = \sin\theta$. $\cos^{-1} x = \frac{\pi}{2} - \theta$.

$$\left(\frac{\pi}{2} - \theta\right) - 2\theta = \cos^{-1}(2\sin\theta)$$

$$\frac{\pi}{2} - 3\theta = \cos^{-1}(2\sin\theta)$$

$$\cos\left(\frac{\pi}{2} - 3\theta\right) = 2\sin\theta$$

$$\sin 3\theta = 2\sin\theta$$

$$3\sin\theta - 4\sin^3\theta = 2\sin\theta$$

$$\sin\theta(1 - 4\sin^2\theta) = 0 \tag{29}$$

$x = 0$ or $x = \pm 1/2$. Checking original eq: $x = 0$: $\pi/2 - 0 = \pi/2$. Correct. $x = 1/2$: $\pi/3 - 2(\pi/6) = 0$. $\cos^{-1}(1) = 0$. Correct. $x = -1/2$: $2\pi/3 - 2(-\pi/6) = \pi$. $\cos^{-1}(-1) = \pi$. Correct. Sum of roots: $0 + 1/2 - 1/2 = 0$.

17. Set $A = \{x \geq 0 : \tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}\}$:

(A) Is an empty set (B) Contains more than two elements
(C) Contains two elements (D) Is a singleton

Answer: (D)
Solution:

$$\frac{5x}{1 - 6x^2} = 1$$

$$6x^2 + 5x - 1 = 0$$

$$(6x - 1)(x + 1) = 0 \tag{30}$$

Roots $1/6, -1$. Since $x \geq 0$, only $x = 1/6$. Singleton set.

18. The sum of the infinite series $\cot^{-1}\left(\frac{7}{4}\right) + \cot^{-1}\left(\frac{19}{4}\right) + \cot^{-1}\left(\frac{39}{4}\right) + \ldots$ is:

(A) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{2}\right)$ (B) $\frac{\pi}{2} - \cot^{-1}\left(\frac{1}{2}\right)$
(C) $\frac{\pi}{2} + \cot^{-1}\left(\frac{1}{2}\right)$ (D) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{2}\right)$

Answer: (D)
Solution:

$$\begin{aligned}T_n &= \cot^{-1}\left(\frac{4n^2+3}{4}\right) = \tan^{-1}\left(\frac{4}{4n^2+3}\right)\\ &= \tan^{-1}\left(\frac{1}{n^2+3/4}\right) = \tan^{-1}\left(\frac{1}{1+(n^2-1/4)}\right)\\ &= \tan^{-1}\left(\frac{(n+0.5)-(n-0.5)}{1+(n+0.5)(n-0.5)}\right)\\ &= \tan^{-1}(n+0.5) - \tan^{-1}(n-0.5) \qquad (31)\end{aligned}$$

Sum telescopes to

$$\begin{aligned}S_\infty &= \lim_{n\to\infty} \tan^{-1}(n+0.5) - \tan^{-1}(0.5)\\ &= \frac{\pi}{2} - \tan^{-1}(1/2) \qquad (32)\end{aligned}$$

19. cosec $\left[2\ \cot^{-1}(5) + \cos^{-1}\left(\frac{4}{5}\right)\right]$ is equal to:

(A) $\frac{65}{56}$ (B) $\frac{75}{56}$ (C) $\frac{65}{33}$ (D) $\frac{56}{33}$

Answer: (A)
Solution: Term $= 2\tan^{-1}(1/5) + \tan^{-1}(3/4)$.

$$2\tan^{-1}(1/5) = \tan^{-1}\left(\frac{2/5}{1-1/25}\right) = \tan^{-1}(5/12) \qquad (33)$$

$$\begin{aligned}\text{Sum} &= \tan^{-1}(5/12) + \tan^{-1}(3/4)\\ &= \tan^{-1}\left(\frac{5/12+3/4}{1-15/48}\right) = \tan^{-1}\left(\frac{56}{33}\right) \qquad (34)\end{aligned}$$

$$\operatorname{cosec}\left(\tan^{-1}(56/33)\right) = \frac{\sqrt{56^2+33^2}}{56} = \frac{65}{56} \qquad (35)$$

20. The value of $\lim_{n\to\infty} 6\ \tan\left\{\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{r^2+3r+3}\right)\right\}$ is equal to

(A) 1 (B) 2 (C) 3 (D) 6

Answer: (C)
Solution:

$$\begin{aligned}\text{Term} &= \tan^{-1}\left(\frac{1}{1+(r+1)(r+2)}\right)\\ &= \tan^{-1}(r+2) - \tan^{-1}(r+1) \qquad (36)\end{aligned}$$

Sum telescopes to

$$\tan^{-1}(n+2) - \tan^{-1}(2) \qquad (37)$$

Limit $n \to \infty$:

$$\frac{\pi}{2} - \tan^{-1}(2) = \cot^{-1}(2) = \tan^{-1}(1/2) \qquad (38)$$

$$\text{Value} = 6\tan\left(\tan^{-1}(1/2)\right) = 6(1/2) = 3 \qquad (39)$$

21. Let $x = \sin\left(2\ \tan^{-1}\alpha\right)$ and $y = \sin\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right)$. If $S = \{\alpha \in \mathbb{R} : y^2 = 1 - x\}$, then $\sum_{\alpha\in S} 16\alpha^3$ is equal to:

Answer: 130
Solution: $y = \sin\left(\frac{1}{2}\tan^{-1}(4/3)\right)$. Let $\tan^{-1}(4/3) = \theta$. $\cos\theta = 3/5$.

$$y = \sqrt{\frac{1-\cos\theta}{2}} = \frac{1}{\sqrt{5}} \tag{40}$$

$y^2 = 1/5$. Given $y^2 = 1 - x \implies x = 4/5$.

$$x = \frac{2\alpha}{1+\alpha^2} = \frac{4}{5} \implies 2\alpha^2 - 5\alpha + 2 = 0 \implies \alpha \in \{2, 1/2\} \tag{41}$$

$$\sum 16\alpha^3 = 16(8 + 1/8) = 128 + 2 = 130 \tag{42}$$

22. For $k \in \mathbb{R}$, let the solutions of the equation $\cos\left(\sin^{-1}(x\ \cot(\tan^{-1}(\cos(\sin^{-1}x))))\right) = k$, $0 < k < 1$, be $\sin^{-1}\left(\frac{2}{\alpha^2}\right)$ and $\sin^{-1}(-1)$. If the sum of the roots of the equation $kx^2 + \sqrt{1-k^2}x - k = 0$ is b, then $\frac{b}{k^2}$ is equal to:

Answer: 12
Solution: Simplifying nested ITF:

$$\cos(\sin^{-1}x) = \sqrt{1-x^2} \tag{43}$$

$$\cot\left(\tan^{-1}\sqrt{1-x^2}\right) = \frac{1}{\sqrt{1-x^2}} \tag{44}$$

Arg becomes $\frac{x}{\sqrt{1-x^2}}$.

$$\cos\left(\sin^{-1}\frac{x}{\sqrt{1-x^2}}\right) = \sqrt{1 - \frac{x^2}{1-x^2}} = \sqrt{\frac{1-2x^2}{1-x^2}} \tag{45}$$

This equals k. Eq roots α, β imply $\beta = -\alpha$. Roots are $\alpha, -\alpha$. Quadratic roots: $\frac{2}{\alpha^2}$ and -1. Product:

$$\frac{-2}{\alpha^2} = -5 \implies \alpha^2 = \frac{2}{5} \tag{46}$$

Then $k = \sqrt{\frac{1-4/5}{1-2/5}} = \sqrt{1/3}$. $k^2 = 1/3$. Sum of roots $= b = 5 - 1 = 4$.

$$b/k^2 = 4/(1/3) = 12 \tag{47}$$

23. Let $f(x)$ be a function such that $f(x+y) = f(x)\cdot f(y)$ for all $x, y \in \mathbb{N}$. If $f(1) = 3$ and $\sum_{k=1}^{n} f(k) = 3279$, then value of n is:

Answer: 7
Solution: $f(x) = 3^x$. Sum is G.P.

$$\begin{aligned} 3 + 3^2 + \cdots + 3^n &= \frac{3(3^n-1)}{2} = 3279 \\ 3^n - 1 &= 2186 \\ 3^n &= 2187 \\ n &= 7 \end{aligned} \tag{48}$$

24. Number of functions $f : \{1, 2, \ldots, 100\} \to \{0, 1\}$ that assign 1 to exactly one of the positive integers less than or equal to 98, is equal to:

Answer: 392
Solution: Choose 1 input from first 98 to map to 1: $\binom{98}{1} = 98$ ways. The other 97 inputs map to 0 (1 way). Remaining inputs $\{99, 100\}$ can map to 0 or 1 freely ($2 \times 2 = 4$ ways).

$$\text{Total} = 98 \times 4 = 392 \tag{49}$$

25. Let $f(x)$ and $g(x)$ be polynomials of degree 2 and 1 respectively. If $f(1) = 8$, $f(2) = 25$ and $f(3) = 52$ and $g(1) = 1$, $g(2) = 3$, then the value of $f(2) + g(2)$ is:

Answer: 28
Solution:

$$f(2) = 25$$
$$g(2) = 3$$
$$\text{Sum} = 28 \tag{50}$$