

**Topic: Vector Algebra**

Mathematics

Topic-wise JEE Main PYQ

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Topic 1: Dot Product Identities**Q1. JEE Main 2026**For three unit vectors $\vec{a}, \vec{b}, \vec{c}$ satisfying
 $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 = 9$ and $|2\vec{a} + k\vec{b} + k\vec{c}| = 3$,
 the positive value of k is :

- (A) 4 (B) 5 (C) 6 (D) 3

Q2. JEE Main 2026
 If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 2$ and $|\vec{b}| = 3$, then
 the maximum value of $3|(3\vec{a} + 2\vec{b})| + 4|(3\vec{a} - 2\vec{b})|$ is :

- (A) 30 (B) 36 (C) 60 (D) 72

Q3. JEE Main 2023
 Let $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$, and $\vec{b}$ and $\vec{c}$ be two nonzero vectors such that
 $|\vec{a} + \vec{b} + \vec{c}| = |\vec{a} + \vec{b} - \vec{c}|$ and $\vec{b} \cdot \vec{c} = 0$. Consider the following
 two statements:

- (A)
- $|\vec{a} + \lambda\vec{c}| \geq |\vec{a}|$
- for all
- $\lambda \in \mathbb{R}$
- .
-
- (B)
- $\vec{a}$
- and
- $\vec{c}$
- are always parallel.

Then,

- (A) only (B) is correct
-
- (B) both (A) and (B) are correct
-
- (C) only (A) is correct
-
- (D) neither (A) nor (B) is correct

Q4. JEE Main 2021
 Let $\vec{a}, \vec{b}, \vec{c}$ be three mutually perpendicular vectors of the
 same magnitude and equally inclined at an angle θ , with the
 vector $\vec{a} + \vec{b} + \vec{c}$. Then $36\cos^2 2\theta$ is equal to _____.
Q5. JEE Main 2021Let $\vec{a}$ and $\vec{b}$ be two vectors
 such that $|2\vec{a} + 3\vec{b}| = |3\vec{a} + \vec{b}|$ and the angle between $\vec{a}$ and
 $\vec{b}$ is 60° . If $\frac{1}{8}\vec{a}$ is a unit vector, then $|\vec{b}|$ is equal to :

- (A) 4 (B) 6 (C) 5 (D) 8

Q6. JEE Main 2020If $\vec{x}$ and $\vec{y}$ be two non-zero vectors such that
 $|\vec{x} + \vec{y}| = |\vec{x}|$ and $2\vec{x} + \lambda\vec{y}$ is perpendicular to $\vec{y}$,
 then the value of λ is _____.
Q7. JEE Main 2020Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that

$$|\vec{a} - \vec{b}|^2 + |\vec{a} - \vec{c}|^2 = 8.$$

Then $|\vec{a} + 2\vec{b}|^2 + |\vec{a} + 2\vec{c}|^2$ is equal to _____.**Q8. JEE Main 2003** $\vec{a}, \vec{b}, \vec{c}$ are 3 vectors, such that

$$\vec{a} + \vec{b} + \vec{c} = 0, |\vec{a}| = 1, |\vec{b}| = 2, |\vec{c}| = 3,$$

then $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ is equal to :

- (A) 1
-
- (B) 0
-
- (C) -7
-
- (D) 7

Q9. JEE Main 2002
 If $|\vec{a}| = 5, |\vec{b}| = 4, |\vec{c}| = 3$ thus what will be the value of
 $|\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}|$, given that $\vec{a} + \vec{b} + \vec{c} = 0$:

- (A) 25 (B) 50 (C) -25 (D) -50

Topic 2: Dot Product Angle**Q10. JEE Main 2026**
 Let a vector $\vec{a} = \sqrt{2}\hat{i} - \hat{j} + \lambda\hat{k}, \lambda > 0$, make an obtuse angle
 with the vector $\vec{b} = -\lambda^2\hat{i} + 4\sqrt{2}\hat{j} + 4\sqrt{2}\hat{k}$ and an angle $\theta, \frac{\pi}{6} < \theta < \frac{\pi}{2}$, with the positive z -axis. If the set of all possible values
 of λ is $(\alpha, \beta) - \{\gamma\}$, then $\alpha + \beta + \gamma$ is equal to _____.
Q11. JEE Main 2026
 Let the vectors $\vec{a} = -\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} + \hat{k}$. For some
 $\lambda, \mu \in \mathbb{R}$, let $\vec{c} = \lambda\vec{a} + \mu\vec{b}$.

 If $\vec{c} \cdot (3\hat{i} - 6\hat{j} + 2\hat{k}) = 10$ and $\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = -2$, then $|\vec{c}|^2$ is
 equal to:

- (A) 8 (B) 12 (C) 14 (D) 15

Q12. JEE Main 2025
 Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that the angle between
 them is $\frac{\pi}{3}$. If $\lambda\vec{a} + 2\vec{b}$ and $3\vec{a} - \lambda\vec{b}$ are perpendicular to each
 other, then the number of values of λ in $[-1, 3]$ is :

- (A) 1 (B) 3 (C) 2 (D) 0

Q13. JEE Main 2024

Let a unit vector $\hat{u} = x\hat{i} + y\hat{j} + z\hat{k}$ make angles $\frac{\pi}{2}$, $\frac{\pi}{3}$ and $\frac{2\pi}{3}$ with the vectors $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}$, $\frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$ and $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$ respectively. If $\vec{v} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$ then $|\hat{u} - \vec{v}|^2$ is equal to

- (A) $\frac{11}{2}$ (B) $\frac{5}{2}$
(C) 7 (D) 9

Q14. JEE Main 2024

The least positive integral value of α , for which the angle between the vectors $\alpha\hat{i} - 2\hat{j} + 2\hat{k}$ and $\alpha\hat{i} + 2\alpha\hat{j} - 2\hat{k}$ is acute, is _____.

Q15. JEE Main 2024

Let a unit vector which makes an angle of 60° with $2\hat{i} + 2\hat{j} - \hat{k}$ and an angle of 45° with $\hat{i} - \hat{k}$ be $\vec{C}$. Then $\vec{C} + (-\frac{1}{2}\hat{i} + \frac{1}{3\sqrt{2}}\hat{j} - \frac{\sqrt{2}}{3}\hat{k})$ is:

- (A) $-\frac{\sqrt{2}}{3}\hat{i} + \frac{\sqrt{2}}{3}\hat{j} + (\frac{1}{2} + \frac{2\sqrt{2}}{3})\hat{k}$
(B) $(\frac{1}{\sqrt{3}} + \frac{1}{2})\hat{i} + (\frac{1}{\sqrt{3}} - \frac{1}{3\sqrt{2}})\hat{j} + (\frac{1}{\sqrt{3}} + \frac{\sqrt{2}}{3})\hat{k}$
(C) $\frac{\sqrt{2}}{3}\hat{i} - \frac{1}{2}\hat{k}$
(D) $\frac{\sqrt{2}}{3}\hat{i} + \frac{1}{3\sqrt{2}}\hat{j} - \frac{1}{2}\hat{k}$

Q16. JEE Main 2024

For $\lambda > 0$, let θ be the angle between the vectors $\vec{a} = \hat{i} + \lambda\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$. If the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are mutually perpendicular, then the value of $(14 \cos \theta)^2$ is equal to

- (A) 25 (B) 50 (C) 20 (D) 40

Q17. JEE Main 2024

Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + 3\hat{j} - 5\hat{k}$ and $\vec{c} = 3\hat{i} - \hat{j} + \lambda\hat{k}$ be three vectors. Let $\vec{r}$ be a unit vector along $\vec{b} + \vec{c}$. If $\vec{r} \cdot \vec{a} = 3$, then 3λ is equal to:

- (A) 21 (B) 25 (C) 27 (D) 30

Q18. JEE Main 2024

The set of all α , for which the vectors $\vec{a} = \alpha\hat{i} + 6\hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} - 2\alpha\hat{k}$ are inclined at an obtuse angle for all $t \in \mathbb{R}$, is

- (A) $[0, 1)$ (B) $(-\frac{4}{3}, 0]$
(C) $(-2, 0]$ (D) $(-\frac{4}{3}, 1)$

Q19. JEE Main 2022

Let S be the set of all $a \in \mathbb{R}$ for which the angle between the vectors $\vec{u} = a(\log_e b)\hat{i} - 6\hat{j} + 3\hat{k}$ and $\vec{v} = (\log_e b)\hat{i} + 2\hat{j} + 2a(\log_e b)\hat{k}$, ($b > 1$) is acute. Then S is equal to:

- (A) $(-\infty, -\frac{4}{3})$ (B) Φ
(C) $(-\frac{4}{3}, 0)$ (D) $(\frac{12}{7}, \infty)$

Q20. JEE Main 2021

If $(\vec{a} + 3\vec{b})$ is perpendicular to $(7\vec{a} - 5\vec{b})$ and $(\vec{a} - 4\vec{b})$ is perpendicular to $(7\vec{a} - 2\vec{b})$, then the angle between $\vec{a}$ and $\vec{b}$ (in degrees) is _____.

Q21. JEE Main 2019

Let $\vec{a} = 2\hat{i} + \lambda_1\hat{j} + 3\hat{k}$, $\vec{b} = 4\hat{i} + (3 - \lambda_2)\hat{j} + 6\hat{k}$, and $\vec{c} = 3\hat{i} + 6\hat{j} + (\lambda_3 - 1)\hat{k}$ be three vectors such that $\vec{b} = 2\vec{a}$ and $\vec{a}$ is perpendicular to $\vec{c}$. Then a possible value of $(\lambda_1, \lambda_2, \lambda_3)$ is:

- (A) (1, 5, 1)
(B) (1, 3, 1)
(C) $(-\frac{1}{2}, 4, 0)$
(D) $(\frac{1}{2}, 4, -2)$

Q22. JEE Main 2012

Let $\vec{a}$ and $\vec{b}$ be two unit vectors. If the vectors $\vec{c} = \vec{a} + 2\vec{b}$ and $\vec{d} = 5\vec{a} - 4\vec{b}$ are perpendicular to each other, then the angle between $\vec{a}$ and $\vec{b}$ is:

- (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

Q23. JEE Main 2010

If the vectors $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} + \hat{k}$ and $\vec{c} = \lambda\hat{i} + \hat{j} + \mu\hat{k}$ are mutually orthogonal, then (λ, μ) is equal to:

- (A) (2, -3) (B) (-2, 3) (C) (3, -2) (D) (-3, 2)

Topic 3: Dot Product Projection**Q24. JEE Main 2026**

Let $\vec{AB} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{AD} = \hat{i} + 2\hat{j} + \lambda\hat{k}$, $\lambda \in \mathbb{R}$. Let the projection of the vector $\vec{v} = \hat{i} + \hat{j} + \hat{k}$ on the diagonal $\vec{AC}$ of the parallelogram ABCD be of length one unit. If α, β , where $\alpha > \beta$, be the roots of the equation $\lambda^2 x^2 - 6\lambda x + 5 = 0$, then $2\alpha - \beta$ is equal to

- (A) 3 (B) 6 (C) 4 (D) 1

Q25. JEE Main 2026

Let $\vec{a} = 2\hat{i} - \hat{j} - \hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} + \hat{j} + 3\hat{k}$. Let $\vec{v}$ be the vector in the plane of the vectors $\vec{a}$ and $\vec{b}$, such that the length of its projection on the vector $\vec{c}$ is $\frac{1}{\sqrt{14}}$. Then $|\vec{v}|$ is equal to

- (A) $\frac{\sqrt{35}}{2}$ (B) 13 (C) $\frac{\sqrt{21}}{2}$ (D) 7

Q26. JEE Main 2025

If the components of $\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ along and perpendicular to $\vec{b} = 3\hat{i} + \hat{j} - \hat{k}$ respectively, are $\frac{16}{11}(3\hat{i} + \hat{j} - \hat{k})$ and $\frac{1}{11}(-4\hat{i} - 5\hat{j} - 17\hat{k})$, then $\alpha^2 + \beta^2 + \gamma^2$ is equal to:

- (A) 16 (B) 23 (C) 26 (D) 18

Q27. JEE Main 2025

If $\vec{a}$ is a nonzero vector such that its projections on the vectors $2\hat{i} - \hat{j} + 2\hat{k}$, $\hat{i} + 2\hat{j} - 2\hat{k}$ and $\hat{k}$ are equal, then a unit vector along $\vec{a}$ is:

- (A) $\frac{1}{\sqrt{155}}(-7\hat{i} + 9\hat{j} + 5\hat{k})$
(B) $\frac{1}{\sqrt{155}}(-7\hat{i} + 9\hat{j} - 5\hat{k})$
(C) $\frac{1}{\sqrt{155}}(7\hat{i} + 9\hat{j} - 5\hat{k})$
(D) $\frac{1}{\sqrt{155}}(7\hat{i} + 9\hat{j} + 5\hat{k})$

Q28. JEE Main 2025

Consider two vectors

$\vec{u} = 3\hat{i} - \hat{j}$ and $\vec{v} = 2\hat{i} + \hat{j} - \lambda\hat{k}$, $\lambda > 0$. The angle between them is given by $\cos^{-1}(\frac{\sqrt{5}}{2\sqrt{7}})$.

Let $\vec{v} = \vec{v}_1 + \vec{v}_2$, where $\vec{v}_1$ is parallel to $\vec{u}$ and $\vec{v}_2$ is perpendicular to $\vec{u}$. Then the value $|\vec{v}_1|^2 + |\vec{v}_2|^2$ is equal to

- (A) $\frac{23}{2}$ (B) $\frac{25}{2}$
(C) 10 (D) 14

Q29. JEE Main 2023

Let $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} + 5\hat{k}$ be two vectors. Then which one of the following statements is TRUE ?

- (A) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-13}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\vec{b}$.
(B) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{13}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\vec{b}$.
(C) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{13}{\sqrt{35}}$ and the direction of the projection vector is same as of $\vec{b}$.
(D) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-13}{\sqrt{35}}$ and the direction of the projection vector is same as of $\vec{b}$.

Q30. JEE Main 2023

Let $\vec{a} = 4\hat{i} + 3\hat{j} + 5\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 4\hat{k}$. Let $\vec{\beta}_1$ be parallel to $\vec{a}$ and $\vec{\beta}_2$ be perpendicular to $\vec{a}$. If $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, then the value of $5\vec{\beta}_2 \cdot (\hat{i} + \hat{j} + \hat{k})$ is :

- (A) 9 (B) 7 (C) 6 (D) 11

Q31. JEE Main 2021

In a triangle ABC, if $|\vec{BC}| = 3$, $|\vec{CA}| = 5$ and $|\vec{AB}| = 7$, then the projection of the vector $\vec{BA}$ on $\vec{BC}$ is equal to :

- (A) $\frac{19}{2}$ (B) $\frac{13}{2}$ (C) $\frac{11}{2}$ (D) $\frac{15}{2}$

Q32. JEE Main 2021

If the projection of the vector $\hat{i} + 2\hat{j} + \hat{k}$ on the sum of the two vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $-\hat{i} + 2\hat{j} + 3\hat{k}$ is 1, then λ is equal to _____.

Q33. JEE Main 2021

In a triangle ABC, if $|\vec{BC}| = 8$, $|\vec{CA}| = 7$, $|\vec{AB}| = 10$, then the projection of the vector $\vec{AB}$ on $\vec{AC}$ is equal to :

- (A) $\frac{25}{4}$ (B) $\frac{127}{20}$ (C) $\frac{85}{14}$ (D) $\frac{115}{16}$

Q34. JEE Main 2020

Let the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ be such that

$|\vec{a}| = 2$, $|\vec{b}| = 4$ and $|\vec{c}| = 4$. If the projection of

$\vec{b}$ on $\vec{a}$ is equal to the projection of $\vec{c}$ on $\vec{a}$ and $\vec{b}$ is perpendicular to $\vec{c}$,

then the value of

$|\vec{a} + \vec{b} - \vec{c}|$ is _____.

Q35. JEE Main 2019

Let $\vec{a} = 3\hat{i} + \hat{j}$ and $\vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$. If $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{a}$ and $\vec{\beta}_2$ is perpendicular to $\vec{a}$, then $\vec{\beta}_1 \times \vec{\beta}_2$ is equal to

- (A) $3\hat{i} - 9\hat{j} - 5\hat{k}$
(B) $\frac{1}{2}(-3\hat{i} + 9\hat{j} + 5\hat{k})$
(C) $-3\hat{i} + 9\hat{j} + 5\hat{k}$
(D) $\frac{1}{2}(3\hat{i} - 9\hat{j} + 5\hat{k})$

Q36. JEE Main 2019

The distance of the point having position vector $-\hat{i} + 2\hat{j} + 6\hat{k}$ from the straight line passing through the point $(2, 3, -4)$ and parallel to the vector, $6\hat{i} + 3\hat{j} - 4\hat{k}$ is :

- (A) 6
(B) 7
(C) $2\sqrt{13}$
(D) $4\sqrt{3}$

Q37. JEE Main 2019

Let $\vec{a} = \hat{i} + \hat{j} + \sqrt{2}\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + \sqrt{2}\hat{k}$, $\vec{c} = 5\hat{i} + \hat{j} + \sqrt{2}\hat{k}$ be three vectors such that the projection vector of $\vec{b}$ on $\vec{a}$ is $\vec{a}$.

If $\vec{a} + \vec{b}$ is perpendicular to $\vec{c}$, then $|\vec{b}|$ is equal to :

- (A) $\sqrt{32}$ (B) 6 (C) $\sqrt{22}$ (D) 4

Q38. JEE Main 2017

If the vector $\vec{b} = 3\hat{j} + 4\hat{k}$ is written as the sum of a vector $\vec{b}_1$, parallel to $\vec{a} = \hat{i} + \hat{j}$ and a vector $\vec{b}_2$, perpendicular to $\vec{a}$, then $\vec{b}_1 \times \vec{b}_2$ is equal to :

- (A) $-3\hat{i} + 3\hat{j} - 9\hat{k}$
(B) $6\hat{i} - 6\hat{j} + \frac{9}{2}\hat{k}$
(C) $-6\hat{i} + 6\hat{j} - \frac{9}{2}\hat{k}$
(D) $3\hat{i} - 3\hat{j} + 9\hat{k}$

Q39. JEE Main 2012

Let ABCD be a parallelogram such that $\vec{AB} = \vec{q}$, $\vec{AD} = \vec{p}$ and $\angle BAD$ be an acute angle. If $\vec{r}$ is the vector that coincide with the altitude directed from the vertex B to the side AD, then $\vec{r}$ is given by :

- (A) $\vec{r} = 3\vec{q} - \frac{3(\vec{p} \cdot \vec{q})}{(\vec{p} \cdot \vec{p})}\vec{p}$
(B) $\vec{r} = -\vec{q} + \frac{(\vec{p} \cdot \vec{q})}{(\vec{p} \cdot \vec{p})}\vec{p}$
(C) $\vec{r} = \vec{q} - \frac{(\vec{p} \cdot \vec{q})}{(\vec{p} \cdot \vec{p})}\vec{p}$
(D) $\vec{r} = -3\vec{q} - \frac{3(\vec{p} \cdot \vec{q})}{(\vec{p} \cdot \vec{p})}\vec{p}$

Q40. JEE Main 2004

Let $\vec{u}$, $\vec{v}$, $\vec{w}$ be such that $|\vec{u}| = 1$, $|\vec{v}| = 2$, $|\vec{w}| = 3$. If the projection $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and $\vec{v}$, $\vec{w}$ are perpendicular to each other then $|\vec{u} - \vec{v} + \vec{w}|$ equals :

- (A) 14 (B) $\sqrt{7}$ (C) $\sqrt{14}$ (D) 2

Topic 4: Parallel / Collinear Vectors

Q41. JEE Main 2024

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors such that $\vec{b}$ and $\vec{c}$ are non-collinear. If $\vec{a} + 5\vec{b}$ is collinear with $\vec{c}$, $\vec{b} + 6\vec{c}$ is collinear with $\vec{a}$ and $\vec{a} + \alpha\vec{b} + \beta\vec{c} = \vec{0}$, then $\alpha + \beta$ is equal to

- (A) 30 (B) -30 (C) -25 (D) 35

Q42. JEE Main 2023

If the points with position vectors $\alpha\hat{i} + 10\hat{j} + 13\hat{k}$, $6\hat{i} + 11\hat{j} + 11\hat{k}$, $\frac{9}{2}\hat{i} + \beta\hat{j} - 8\hat{k}$ are collinear, then $(19\alpha - 6\beta)^2$ is equal to :

- (A) 16 (B) 49 (C) 36 (D) 25

Q43. JEE Main 2022

Let A, B, C be three points whose position vectors respectively are

$$\vec{a} = \hat{i} + 4\hat{j} + 3\hat{k}$$

$$\vec{b} = 2\hat{i} + \alpha\hat{j} + 4\hat{k}, \alpha \in R$$

$$\vec{c} = 3\hat{i} - 2\hat{j} + 5\hat{k}$$

If α is the smallest positive integer for which $\vec{a}$, $\vec{b}$, $\vec{c}$ are noncollinear, then the length of the median, in $\triangle ABC$, through A is :

- (A) $\frac{\sqrt{82}}{2}$ (B) $\frac{\sqrt{62}}{2}$ (C) $\frac{\sqrt{69}}{2}$ (D) $\frac{\sqrt{66}}{2}$

Q44. JEE Main 2021

If vectors $\vec{a}_1 = x\hat{i} - \hat{j} + \hat{k}$ and $\vec{a}_2 = \hat{i} + y\hat{j} + z\hat{k}$ are collinear, then a possible unit vector parallel to the vector $x\hat{i} + y\hat{j} + z\hat{k}$ is :

- (A) $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$
 (B) $\frac{1}{\sqrt{2}}(-\hat{j} + \hat{k})$
 (C) $\frac{1}{\sqrt{2}}(\hat{i} - \hat{j})$
 (D) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} - \hat{k})$

Q45. JEE Main 2019

If $\vec{a} = (\lambda - 2)\vec{a} + \vec{b}$ and $\vec{\beta} = (4\lambda - 2)\vec{a} + 3\vec{b}$ be two given vectors $\vec{a}$ and $\vec{b}$ are non-collinear. The value of λ for which vectors $\vec{a}$ and $\vec{\beta}$ are collinear, is -

- (A) 4 (B) 3 (C) -3 (D) -4

Q46. JEE Main 2011

Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three non-zero vectors which are pairwise non-collinear. If $\vec{a} + 3\vec{b}$ is collinear with $\vec{c}$ and $\vec{b} + 2\vec{c}$ is collinear with $\vec{a}$, then $\vec{a} + \vec{b} + 6\vec{c}$ is :

- (A) $\vec{a} + \vec{c}$ (B) $\vec{c}$ (C) $\vec{a}$ (D) $\vec{0}$

Q47. JEE Main 2004

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of these are collinear. If the vector $\vec{a} + 2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b} + 3\vec{c}$ is collinear with $\vec{a}$ (λ being some non-zero scalar) then $\vec{a} + 2\vec{b} + 6\vec{c}$ equals to :

- (A) $\vec{0}$ (B) $\lambda\vec{b}$ (C) $\lambda\vec{c}$ (D) $\lambda\vec{a}$

Topic 5: Theorem in a Plane

Q48. JEE Main 2025

Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$. Let $\hat{e}$ be a unit vector in the plane of the vectors $\vec{a}$ and $\vec{b}$ and be perpendicular to $\vec{a}$. Then such a vector $\hat{e}$ is:

- (A) $\frac{1}{\sqrt{2}}(-\hat{i} + \hat{k})$
 (B) $\frac{1}{\sqrt{3}}(\hat{j} - 2\hat{k})$
 (C) $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$
 (D) $\frac{1}{\sqrt{3}}(-\hat{i} + \hat{j} - \hat{k})$

Q49. JEE Main 2021

Let $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$. Let a vector $\vec{v}$ be in the plane containing $\vec{a}$ and $\vec{b}$. If $\vec{v}$ is perpendicular to the vector $3\hat{i} + 2\hat{j} - \hat{k}$ and its projection on $\vec{a}$ is 19 units, then $|2\vec{v}|^2$ is equal to _____.

Q50. JEE Main 2021

Let $\vec{x}$ be a vector in the plane containing vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$. If the vector $\vec{x}$ is perpendicular to $(3\hat{i} + 2\hat{j} - \hat{k})$ and its projection on $\vec{a}$ is $\frac{17\sqrt{6}}{2}$, then the value of $|\vec{x}|^2$ is equal to _____.

Q51. JEE Main 2018

Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u} \cdot \vec{b} = 24$, then $|\vec{u}|^2$ is equal to

- (A) 336 (B) 315
 (C) 256 (D) 84

Topic 6: Cross Product Calculation

Q52. JEE Main 2024

Let $\vec{a} = 3\hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c}$ be a vector such that $(\vec{a} + \vec{b}) \times \vec{c} = 2(\vec{a} \times \vec{b}) + 24\hat{j} - 6\hat{k}$ and $(\vec{a} - \vec{b} + \hat{i}) \cdot \vec{c} = -3$. Then $|\vec{c}|^2$ is equal to _____.

Q53. JEE Main 2025

Let $\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = \vec{a} \times (\hat{i} - 2\hat{k})$ and $\vec{c} = \vec{b} \times \hat{k}$. Then the projection of $\vec{c} - 2\hat{j}$ on $\vec{a}$ is :

- (A) $2\sqrt{7}$ (B) $3\sqrt{7}$ (C) $\sqrt{14}$ (D) $2\sqrt{14}$

Q54. JEE Main 2024

Let $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{b} = ((\vec{a} \times (\hat{i} + \hat{j})) \times \hat{i}) \times \hat{i}$. Then the square of the projection of $\vec{a}$ on $\vec{b}$ is:

- (A) $\frac{1}{3}$ (B) $\frac{1}{5}$ (C) 2 (D) $\frac{2}{3}$

Q55. JEE Main 2026

Let $\vec{a} = -\hat{i} + 2\hat{j} + 2\hat{k}$, $\vec{b} = 8\hat{i} + 7\hat{j} - 3\hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} \times \vec{c} = \vec{b}$. If $\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = 4$, then $|\vec{a} + \vec{c}|^2$ is equal to :

- (A) 30 (B) 33 (C) 27 (D) 35

Q56. JEE Main 2026

If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{j} - \hat{k}$ and $\vec{c}$ be three vectors such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$, then $\vec{c} \cdot (\vec{a} - 2\vec{b})$ is equal to ____.

Q57. JEE Main 2023

Let $\vec{a} = -\hat{i} - \hat{j} + \hat{k}$, $\vec{a} \cdot \vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{i} - \hat{j}$. Then $\vec{a} - 6\vec{b}$ is equal to :

- (A) $3(\hat{i} + \hat{j} + \hat{k})$ (B) $3(\hat{i} - \hat{j} - \hat{k})$
(C) $3(\hat{i} + \hat{j} - \hat{k})$ (D) $3(\hat{i} - \hat{j} + \hat{k})$

Q58. JEE Main 2022

Let $\vec{a} = \alpha\hat{i} + 3\hat{j} - \hat{k}$, $\vec{b} = 3\hat{i} - \beta\hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + 2\hat{j} - 2\hat{k}$ where $\alpha, \beta \in \mathbb{R}$, be three vectors. If the projection of $\vec{a}$ on $\vec{c}$ is $\frac{10}{3}$ and $\vec{b} \times \vec{c} = -6\hat{i} + 10\hat{j} + 7\hat{k}$, then the value of $\alpha + \beta$ is equal to :

- (A) 3 (B) 4 (C) 5 (D) 6

Q59. JEE Main 2022

Let $\vec{a} = \alpha\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \alpha\hat{k}$, $\alpha > 0$. If the projection of $\vec{a} \times \vec{b}$ on the vector $-\hat{i} + 2\hat{j} - 2\hat{k}$ is 30, then α is equal to :

- (A) $\frac{15}{2}$ (B) 8 (C) $\frac{13}{2}$ (D) 7

Q60. JEE Main 2022

Let $\vec{a} = \alpha\hat{i} + \hat{j} + \beta\hat{k}$ and $\vec{b} = 3\hat{i} - 5\hat{j} + 4\hat{k}$ be two vectors, such that $\vec{a} \times \vec{b} = -\hat{i} + 9\hat{j} + 12\hat{k}$. Then the projection of $\vec{b} - 2\vec{a}$ on $\vec{b} + \vec{a}$ is equal to :

- (A) 2 (B) $\frac{39}{5}$ (C) 9 (D) $\frac{46}{5}$

Q61. JEE Main 2022

Let $\vec{a}$ be a vector which is perpendicular to the vector $3\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k}$. If $\vec{a} \times (2\hat{i} + \hat{k}) = 2\hat{i} - 13\hat{j} - 4\hat{k}$, then the projection of the vector $\vec{a}$ on the vector $2\hat{i} + 2\hat{j} + \hat{k}$ is :

- (A) $\frac{1}{3}$ (B) 1 (C) $\frac{5}{3}$ (D) $\frac{7}{3}$

Q62. JEE Main 2022

Let $\vec{b} = \hat{i} + \hat{j} + \lambda\hat{k}$, $\lambda \in \mathbb{R}$. If $\vec{a}$ is a vector such that $\vec{a} \times \vec{b} = 13\hat{i} - \hat{j} - 4\hat{k}$ and $\vec{a} \cdot \vec{b} + 21 = 0$, then $(\vec{b} - \vec{a}) \cdot (\hat{k} - \hat{j}) + (\vec{b} + \vec{a}) \cdot (\hat{i} - \hat{k})$ is equal to _____.

Q63. JEE Main 2021

Let $\vec{p} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{q} = \hat{i} + 2\hat{j} + \hat{k}$ be two vectors. If a vector $\vec{r} = (\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k})$ is perpendicular to each of the vectors $(\vec{p} + \vec{q})$ and $(\vec{p} - \vec{q})$, and $|\vec{r}| = \sqrt{3}$, then $|\alpha| + |\beta| + |\gamma|$ is equal to _____.

Q64. JEE Main 2021

Let $\vec{c}$ be a vector perpendicular to the vectors, $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$. If $\vec{c} \cdot (\hat{i} + \hat{j} + 3\hat{k}) = 8$ then the value of $\vec{c} \cdot (\vec{a} \times \vec{b})$ is equal to _____.

Q65. JEE Main 2019

Let $\vec{a} = 3\hat{i} + 2\hat{j} + x\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, for some real x. Then $|\vec{a} \times \vec{b}| = r$

is possible if :

- (A) $0 < r < \sqrt{\frac{3}{2}}$
(B) $3\sqrt{\frac{3}{2}} < r < 5\sqrt{\frac{3}{2}}$
(C) $r \geq 5\sqrt{\frac{3}{2}}$
(D) $\sqrt{\frac{3}{2}} < r \leq 3\sqrt{\frac{3}{2}}$

Q66. JEE Main 2019

Let $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ be two vectors. If a vector perpendicular to both the vectors

$\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ has the magnitude 12 then one such vector is :

- (A) $4(2\hat{i} - 2\hat{j} - \hat{k})$
(B) $4(-2\hat{i} - 2\hat{j} + \hat{k})$
(C) $4(2\hat{i} + 2\hat{j} + \hat{k})$
(D) $4(2\hat{i} + 2\hat{j} - \hat{k})$

Q67. JEE Main 2019

Let $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} \times \vec{c} + \vec{b} = \vec{0}$ and $\vec{a} \cdot \vec{c} = 4$, then $|\vec{c}|^2$ is equal to :

- (A) 8 (B) $\frac{19}{2}$ (C) 9 (D) $\frac{17}{2}$

Q68. JEE Main 2018

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{c} = \hat{j} - \hat{k}$ and a vector $\vec{b}$ be such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \cdot \vec{b} = 3$. Then $|\vec{b}|$ equals :

- (A) $\frac{11}{3}$ (B) $\frac{11}{\sqrt{3}}$ (C) $\sqrt{\frac{11}{3}}$ (D) $\frac{\sqrt{11}}{3}$

Q69. JEE Main 2010

Let $\vec{a} = \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. Then the vector $\vec{b}$ satisfying $\vec{a} \times \vec{b} + \vec{c} = \vec{0}$ and $\vec{a} \cdot \vec{b} = 3$:

- (A) $2\hat{i} - \hat{j} + 2\hat{k}$ (B) $\hat{i} - \hat{j} - 2\hat{k}$
(C) $\hat{i} + \hat{j} - 2\hat{k}$ (D) $-\hat{i} + \hat{j} - 2\hat{k}$

Q70. JEE Main 2005

For any vector $\vec{a}$, the value of $(\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2$ is equal to :

- (A) $3\vec{a}^2$ (B) $\vec{a}^2$ (C) $2\vec{a}^2$ (D) $4\vec{a}^2$

Topic 7: Cross Product Properties**Q71. JEE Main 2026**

For a triangle ABC , let $\vec{p} = \vec{BC}$, $\vec{q} = \vec{CA}$ and $\vec{r} = \vec{BA}$. If $|\vec{p}| = 2\sqrt{3}$, $|\vec{q}| = 2$ and $\cos \theta = \frac{1}{\sqrt{3}}$, where θ is the angle between $\vec{p}$ and $\vec{q}$, then $|\vec{p} \times (\vec{q} - 3\vec{r})|^2 + 3|\vec{r}|^2$ is equal to :

- (A) 200 (B) 220 (C) 410 (D) 340

Q72. JEE Main 2026

Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \vec{a} \times \vec{b}$. Let $\vec{d}$ be a vector such that $|\vec{d} - \vec{a}| = \sqrt{11}$, $|\vec{c} \times \vec{d}| = 3$ and the angle between $\vec{c}$ and $\vec{d}$ is $\frac{\pi}{4}$. Then $\vec{a} \cdot \vec{d}$ is equal to

- (A) 0 (B) 1
(C) 3 (D) 11

Q73. JEE Main 2025

Let $\hat{a}$ be a unit vector perpendicular to the vectors $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{c} = 2\hat{i} + 3\hat{j} - \hat{k}$, and $\hat{a}$ makes an angle of $\cos^{-1}(-\frac{1}{3})$ with the vector $\hat{i} + \hat{j} + \hat{k}$. If $\hat{a}$ makes an angle of $\frac{\pi}{3}$ with the vector $\hat{i} + \hat{j} + \hat{k}$, then the value of a is:

- (A) $\sqrt{3}$ (B) $\sqrt{6}$
(C) $-\sqrt{6}$ (D) $-\sqrt{3}$

Q74. JEE Main 2024

Let $\vec{a} = \hat{i} + \hat{j} + \beta\hat{k}$, $\alpha, \beta \in \mathbb{R}$. Let a vector $\vec{b}$ be such that the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$ and $|\vec{b}|^2 = 6$. If $\vec{a} \cdot \vec{b} = 3\sqrt{2}$, then the value of $(\alpha^2 + \beta^2) |\vec{a} \times \vec{b}|^2$ is equal to

- (A) 85 (B) 90 (C) 75 (D) 95

Q75. JEE Main 2023

Let $|\vec{a}| = 2$, $|\vec{b}| = 3$ and the angle between the vectors $\vec{a}$ and $\vec{b}$ be $\frac{\pi}{4}$. Then $|(\vec{a} + 2\vec{b}) \times (2\vec{a} - 3\vec{b})|^2$ is equal to :

- (A) 441 (B) 482 (C) 841 (D) 882

Q76. JEE Main 2023

Let $\vec{a} = 3\hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} - 3\hat{j} + 3\hat{k}$. If $\vec{b}$ is a vector such that $\vec{a} = \vec{b} \times \vec{c}$ and $|\vec{b}|^2 = 50$, then $|72 - |\vec{b} + \vec{c}|^2|$ is equal to _____.

Q77. JEE Main 2026

Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{c} = \lambda\hat{i} + \hat{j} + \hat{k}$ and $\vec{v} = \vec{a} \times \vec{b}$. If $\vec{v} \cdot \vec{c} = 11$ and the length of the projection of $\vec{b}$ on $\vec{c}$ is p , then $9p^2$ is equal to

- (A) 9 (B) 6 (C) 4 (D) 12

Q78. JEE Main 2023

Let $\vec{a} = 2\hat{i} + 7\hat{j} - \hat{k}$, $\vec{b} = 3\hat{i} + 5\hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + 2\hat{k}$. Let $\vec{d}$ be a vector which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c} \cdot \vec{d} = 12$. Then $(-\hat{i} + \hat{j} - \hat{k}) \cdot (\vec{c} \times \vec{d})$ is equal to :

- (A) 24 (B) 42 (C) 44 (D) 48

Q79. JEE Main 2023

Let O be the origin and the position vector of the point P be $-\hat{i} - 2\hat{j} + 3\hat{k}$. If the position vectors of the points A, B and C are $-2\hat{i} + \hat{j} - 3\hat{k}$, $2\hat{i} + 4\hat{j} - 2\hat{k}$ and $-4\hat{i} + 2\hat{j} - \hat{k}$ respectively, then the projection of the vector $\vec{OP}$ on a vector perpendicular to the vectors $\vec{AB}$ and $\vec{AC}$ is :

- (A) $\frac{7}{3}$ (B) 3 (C) $\frac{10}{3}$ (D) $\frac{8}{3}$

Q80. JEE Main 2023

Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}| = \sqrt{14}$, $|\vec{b}| = \sqrt{6}$ and $|\vec{a} \times \vec{b}| = \sqrt{48}$. Then $(\vec{a} \cdot \vec{b})^2$ is equal to _____.

Q81. JEE Main 2023

Let $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} - 2\hat{k}$ and $\vec{c} = -\hat{i} + 4\hat{j} + 3\hat{k}$. If $\vec{d}$ is a vector perpendicular to both $\vec{b}$ and $\vec{c}$, and $\vec{a} \cdot \vec{d} = 18$, then $|\vec{a} \times \vec{d}|^2$ is equal to :

- (A) 680 (B) 720 (C) 760 (D) 640

Q82. JEE Main 2022

Let $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ and let $\vec{b}$ be a vector such that $\vec{a} \times \vec{b} = 2\hat{i} - \hat{k}$ and $\vec{a} \cdot \vec{b} = 3$. Then the projection of $\vec{b}$ on the vector $\vec{a} - \vec{b}$ is :

- (A) $\frac{2}{\sqrt{21}}$ (B) $2\sqrt{\frac{3}{7}}$
(C) $\frac{2}{3}\sqrt{\frac{7}{3}}$ (D) $\frac{2}{3}$

Q83. JEE Main 2022

Let $\hat{a}$ and $\hat{b}$ be two unit vectors such that the angle between them is $\frac{\pi}{4}$. If θ is the angle between the vectors $(\hat{a} + \hat{b})$ and $(\hat{a} + 2\hat{b} + 2(\hat{a} \times \hat{b}))$, then the value of $164 \cos^2 \theta$ is equal to :

- (A) $90 + 27\sqrt{2}$ (B) $45 + 18\sqrt{2}$
(C) $90 + 3\sqrt{2}$ (D) $54 + 90\sqrt{2}$

Q84. JEE Main 2022

Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + 2|\vec{b}|^2$, $\vec{a} \cdot \vec{b} = 3$ and $|\vec{a} \times \vec{b}|^2 = 75$. Then $|\vec{a}|^2$ is equal to _____.

Q85. JEE Main 2022

Let θ be the angle between the vectors $\vec{a}$ and $\vec{b}$, where $|\vec{a}| = 4$, $|\vec{b}| = 3$ and $\theta \in (\frac{\pi}{4}, \frac{\pi}{3})$. Then $|(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})|^2 + 4(\vec{a} \cdot \vec{b})^2$ is equal to _____.

Q86. JEE Main 2022

Let $\hat{a}$ and $\hat{b}$ be two unit vectors such that $|(\hat{a} + \hat{b}) + 2(\hat{a} \times \hat{b})| = 2$. If $\theta \in (0, \pi)$ is the angle between $\hat{a}$ and $\hat{b}$, then among the statements :

- (S1) : $2|\hat{a} \times \hat{b}| = |\hat{a} - \hat{b}|$
(S2) : The projection of $\hat{a}$ on $(\hat{a} + \hat{b})$ is $\frac{1}{2}$
(A) Only (S1) is true.
(B) Only (S2) is true.
(C) Both (S1) and (S2) are true.
(D) Both (S1) and (S2) are false.

Q87. JEE Main 2022

Let $\hat{a}$, $\hat{b}$ be unit vectors. If $\vec{c}$ be a vector such that the angle between $\hat{a}$ and $\vec{c}$ is $\frac{\pi}{12}$, and $\hat{b} = \vec{c} + 2(\vec{c} \times \hat{a})$, then $|6\vec{c}|^2$ is equal to :

- (A) $6(3 - \sqrt{3})$ (B) $3 + \sqrt{3}$
(C) $6(3 + \sqrt{3})$ (D) $6(\sqrt{3} + 1)$

Q88. JEE Main 2021

If $|\vec{a}| = 2$, $|\vec{b}| = 5$ and $|\vec{a} \times \vec{b}| = 8$, then $|\vec{a} \cdot \vec{b}|$ is equal to :

- (A) 6 (B) 4 (C) 3 (D) 5

Q89. JEE Main 2021

Let $\vec{a} = \hat{i} + 5\hat{j} + a\hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} + \beta\hat{k}$ and $\vec{c} = -\hat{i} + 2\hat{j} - 3\hat{k}$ be three vectors such that, $|\vec{b} \times \vec{c}| = 5\sqrt{3}$ and $\vec{a}$ is perpendicular to $\vec{b}$. Then the greatest amongst the values of $|\vec{a}|^2$ is _____.

Q90. JEE Main 2021

Let $\vec{a}$ and $\vec{b}$ be two non-zero vectors perpendicular to each other and $|\vec{a}| = |\vec{b}|$. If $|\vec{a} \times \vec{b}| = |\vec{a}|$, then the angle between the vectors $(\vec{a} + \vec{b} + (\vec{a} \times \vec{b}))$ and $\vec{a}$ is equal to :

- (A) $\sin^{-1}(\frac{1}{\sqrt{6}})$ (B) $\cos^{-1}(\frac{1}{\sqrt{2}})$
(C) $\sin^{-1}(\frac{1}{\sqrt{3}})$ (D) $\cos^{-1}(\frac{1}{\sqrt{3}})$

Q91. JEE Main 2020

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. If $\lambda = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ and $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$, then the ordered pair, $(\lambda, \vec{d})$ is equal to :

- (A) $(\frac{3}{2}, 3\vec{a} \times \vec{c})$
(B) $(-\frac{3}{2}, 3\vec{c} \times \vec{b})$
(C) $(-\frac{3}{2}, 3\vec{a} \times \vec{b})$
(D) $(\frac{3}{2}, 3\vec{b} \times \vec{c})$

Q92. JEE Main 2018

If $\vec{a}$, $\vec{b}$, and $\vec{c}$ are unit vectors such that $\vec{a} + 2\vec{b} + 2\vec{c} = \vec{0}$, then $|\vec{a} \times \vec{c}|$ is equal to :

- (A) $\frac{\sqrt{15}}{4}$ (B) $\frac{1}{4}$ (C) $\frac{15}{16}$ (D) $\frac{\sqrt{15}}{16}$

Q93. JEE Main 2007

If $\hat{u}$ and $\hat{v}$ are unit vectors and θ is the acute angle between them, then $2\hat{u} \times 3\hat{v}$ is a unit vector for :

- (A) no value of θ
(B) exactly one value of θ
(C) exactly two values of θ
(D) more than two values of θ

Q94. JEE Main 2003

Let $\vec{u} = \hat{i} + \hat{j}$, $\vec{v} = \hat{i} - \hat{j}$ and $\vec{w} = \hat{i} + 2\hat{j} + 3\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{u} \cdot \hat{n} = 0$ and $\vec{v} \cdot \hat{n} = 0$, then $|\vec{w} \cdot \hat{n}|$ is equal to :

- (A) 3 (B) 0 (C) 1 (D) 2

Q95. JEE Main 2002

If $|\vec{a}| = 4$, $|\vec{b}| = 2$ and the angle between $\vec{a}$ and $\vec{b}$ is $\pi/6$ then $(\vec{a} \times \vec{b})^2$ is equal to :

- (A) 48 (B) 16
(C) $\vec{a}$ (D) none of these

Q96. JEE Main 2024

Consider three vectors $\vec{a}, \vec{b}, \vec{c}$. Let $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a} = \vec{b} \times \vec{c}$. If $\alpha \in [0, \frac{\pi}{3}]$ is the angle between the vectors $\vec{b}$ and $\vec{c}$, then the minimum value of $27|\vec{c} - \vec{a}|^2$ is equal to:

- (A) 124 (B) 110 (C) 121 (D) 105

Topic 8: Cross Product Application: Area**Q97. JEE Main 2026**

Let $\vec{a} = 2\hat{i} + 3\hat{j} + 3\hat{k}$ and $\vec{b} = 6\hat{i} + 3\hat{j} + 3\hat{k}$. Then the square of the area of the triangle with adjacent sides determined by the vectors $(2\vec{a} + 3\vec{b})$ and $(\vec{a} - \vec{b})$ is :

- (A) 450 (B) 900 (C) 1800 (D) 2400

Q98. JEE Main 2025

Let the position vectors of the vertices A, B and C of a tetrahedron ABCD be $\hat{i} + 2\hat{j} + \hat{k}$, $\hat{i} + 3\hat{j} - 2\hat{k}$ and $2\hat{i} + \hat{j} - \hat{k}$ respectively. The altitude from the vertex D to the opposite face ABC meets the median line segment through A of the triangle ABC at the point E. If the length of AD is $\frac{\sqrt{110}}{3}$ and the volume of the tetrahedron is $\frac{\sqrt{805}}{6\sqrt{2}}$, then the position vector of E is

- (A) $\frac{1}{6}(7\hat{i} + 12\hat{j} + \hat{k})$
(B) $\frac{1}{12}(7\hat{i} + 4\hat{j} + 3\hat{k})$
(C) $\frac{1}{6}(12\hat{i} + 12\hat{j} + \hat{k})$
(D) $\frac{1}{2}(\hat{i} + 4\hat{j} + 7\hat{k})$

Q99. JEE Main 2023

Let for a triangle ABC,

$$\vec{AB} = -2\hat{i} + \hat{j} + 3\hat{k}$$

$$\vec{CB} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$$

$$\vec{CA} = 4\hat{i} + 3\hat{j} + \delta\hat{k}$$

If $\delta > 0$ and the area of the triangle ABC is $5\sqrt{6}$, then $\vec{CB} \cdot \vec{CA}$ is equal to

- (A) 60 (B) 54 (C) 120 (D) 108

Q100. JEE Main 2025

Let $\vec{c}$ be the projection vector of $\vec{b} = \lambda\hat{i} + 4\hat{k}$, $\lambda > 0$, on the vector $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$. If $|\vec{a} + \vec{c}| = 7$, then the area of the parallelogram formed by the vectors $\vec{b}$ and $\vec{c}$ is _____.

Q101. JEE Main 2024

Let $\vec{a} = 2\hat{i} + \alpha\hat{j} + \hat{k}$, $\vec{b} = -\hat{i} + \hat{k}$, $\vec{c} = \beta\hat{j} - \hat{k}$, where α and β are integers and $\alpha\beta = -6$. Let the values of the ordered pair (α, β) , for which the area of the parallelogram of diagonals $\vec{a} + \vec{b}$ and $\vec{b} + \vec{c}$ is $\frac{\sqrt{21}}{2}$, be (α_1, β_1) and (α_2, β_2) . Then $\alpha_1^2 + \beta_1^2 - \alpha_2\beta_2$ is equal to

- (A) 21 (B) 24 (C) 19 (D) 17

Q102. JEE Main 2024

Let three vectors, $\vec{a} = \alpha\hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 5\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$ form a triangle such that $\vec{c} = \vec{a} - \vec{b}$ and the area of the triangle is $5\sqrt{6}$. If α is a positive real number, then $|\vec{c}|^2$ is equal to:

- (A) 14
(B) 12
(C) 16
(D) 10

Q103. JEE Main 2024

Let $\vec{OA} = 2\vec{a}$, $\vec{OB} = 6\vec{a} + 5\vec{b}$ and $\vec{OC} = 3\vec{b}$, where O is the origin. If the area of the parallelogram with adjacent sides $\vec{OA}$ and $\vec{OC}$ is 15 sq. units, then the area (in sq. units) of the quadrilateral $OACB$ is equal to:

- (A) 32
(B) 38
(C) 35
(D) 40

Q104. JEE Main 2024

Let ABC be a triangle of area $15\sqrt{2}$ and the vectors $\vec{AB} = \hat{i} + 2\hat{j} - 7\hat{k}$, $\vec{BC} = a\hat{i} + b\hat{j} + c\hat{k}$ and $\vec{AC} = 6\hat{i} + d\hat{j} - 2\hat{k}$, $d > 0$. Then the square of the length of the largest side of the triangle ABC is _____.

Q105. JEE Main 2024

If $A(1, -1, 2)$, $B(5, 7, -6)$, $C(3, 4, -10)$ and $D(-1, -4, -2)$ are the vertices of a quadrilateral $ABCD$, then its area is:

- (A) $24\sqrt{7}$ (B) $48\sqrt{7}$ (C) $24\sqrt{29}$ (D) $12\sqrt{29}$

Q106. JEE Main 2023

$A(2, 6, 2)$, $B(-4, 0, \lambda)$, $C(2, 3, -1)$ and $D(4, 5, 0)$, $|\lambda| \leq 5$ are the vertices of a quadrilateral $ABCD$. If its area is 18 square units, then $5 - 6\lambda$ is equal to _____.

Q107. JEE Main 2024

Let $\vec{OA} = \vec{a}$, $\vec{OB} = 12\vec{a} + 4\vec{b}$ and $\vec{OC} = \vec{b}$, where O is the origin. If S is the parallelogram with adjacent sides OA and OC , then $\frac{\text{area of the quadrilateral } OACB}{\text{area of } S}$ is equal to _____.

- (A) 7 (B) 6 (C) 8 (D) 10

Q108. JEE Main 2023

Let PQR be a triangle. The points A , B and C are on the sides QR , RP and PQ respectively such that

$$\frac{QA}{AR} = \frac{RB}{BP} = \frac{PC}{CQ} = \frac{1}{2}. \text{ Then } \frac{\text{Area}(\Delta PQR)}{\text{Area}(\Delta ABC)} \text{ is equal to:}$$

- (A) $\frac{5}{2}$ (B) 4 (C) 2 (D) 3

Q109. JEE Main 2023

The area of the quadrilateral $ABCD$ with vertices $A(2, 1, 1)$, $B(1, 2, 5)$, $C(-2, -3, 5)$ and $D(1, -6, -7)$ is equal to:

- (A) 48 (B) $8\sqrt{38}$ (C) 54 (D) $9\sqrt{38}$

Q110. JEE Main 2022

Let $\vec{a} = \alpha\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = -2\hat{i} + \alpha\hat{j} + \hat{k}$, where $\alpha \in R$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $\sqrt{15(\alpha^2 + 4)}$, then the value of $2|\vec{a}|^2 + (\vec{a} \cdot \vec{b})|\vec{b}|^2$ is equal to:

- (A) 10 (B) 7 (C) 9 (D) 14

Q111. JEE Main 2021

Let $\vec{a} = \hat{i} + \alpha\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - \alpha\hat{j} + \hat{k}$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $8\sqrt{3}$ square units, then $\vec{a} \cdot \vec{b}$ is equal to _____.

Q112. JEE Main 2017

The area (in sq. units) of the parallelogram whose diagonals are along the vectors $8\hat{i} - 6\hat{j}$ and $3\hat{i} + 4\hat{j} - 12\hat{k}$, is:

- (A) 26 (B) 65 (C) 20 (D) 52

Topic 9: Most Repeated Pattern**Q113. JEE Main 2026**

Let $\vec{c}$ and $\vec{d}$ be vectors such that $|\vec{c} + \vec{d}| = \sqrt{29}$ and $\vec{c} \times (2\hat{i} + 3\hat{j} + 4\hat{k}) = (2\hat{i} + 3\hat{j} + 4\hat{k}) \times \vec{d}$. If λ_1, λ_2 ($\lambda_1 > \lambda_2$) are the possible values of $(\vec{c} + \vec{d}) \cdot (-7\hat{i} + 2\hat{j} + 3\hat{k})$, then the equation $K^2x^2 + (K^2 - 5K + \lambda_1)xy + (3K + \frac{\lambda_2}{2})y^2 - 8x + 12y + \lambda_2 = 0$ represents a circle, for K equal to:

- (A) 4 (B) -1 (C) 2 (D) 1

Q114. JEE Main 2026

Let $\vec{a} = \sqrt{7}\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{j} + 2\hat{k}$. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} + \vec{a} \times \vec{b} = \vec{0}$ and $\vec{r} \cdot \vec{a} = 0$, then $|3\vec{r}|^2$ is equal to:

- (A) 44 (B) 54 (C) 86 (D) 132

Q115. JEE Main 2025

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} + 2\hat{j} - \hat{k}$, $\vec{c} = \lambda\hat{j} + \mu\hat{k}$ and $\hat{d}$ be a unit vector such that $\vec{a} \times \hat{d} = \vec{b} \times \hat{d}$ and $\vec{c} \cdot \hat{d} = 1$. If $\vec{c}$ is perpendicular to $\vec{a}$, then $|3\lambda\hat{d} + \mu\vec{c}|^2$ is equal to _____.

Q116. JEE Main 2025

Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} - 3\hat{j} + 3\hat{k}$, $\vec{c} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{d}$ be a vector such that $\vec{b} \times \vec{d} = \vec{c} \times \vec{d}$ and $\vec{a} \cdot \vec{d} = 4$. Then $|(\vec{a} \times \vec{d})|^2$ is equal to _____.

Q117. JEE Main 2023

Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. If a vector $\vec{d}$ satisfies $\vec{d} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{d} \cdot \vec{a} = 24$, then $|\vec{d}|^2$ is equal to:

- (A) 313 (B) 413 (C) 423 (D) 323

Q118. JEE Main 2024

Let $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = 4\hat{i} + \hat{j} + 7\hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} + 4\hat{k}$ be three vectors. If a vectors $\vec{p}$ satisfies $\vec{p} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{p} \cdot \vec{a} = 0$, then $\vec{p} \cdot (\hat{i} - \hat{j} - \hat{k})$ is equal to

- (A) 24 (B) 32 (C) 36 (D) 28

Q119. JEE Main 2024

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = -\hat{i} - 8\hat{j} + 2\hat{k}$ and $\vec{c} = 4\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three vectors such that $\vec{b} \times \vec{a} = \vec{c} \times \vec{a}$. If the angle between the vector $\vec{c}$ and the vector $3\hat{i} + 4\hat{j} + \hat{k}$ is θ , then the greatest integer less than or equal to $\tan^2 \theta$ is _____.

Q120. JEE Main 2024

Between the following two statements:

Statement I : Let $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$. Then the vector $\vec{r}$ satisfying $\vec{a} \times \vec{r} = \vec{a} \times \vec{b}$ and $\vec{a} \cdot \vec{r} = 0$ is of magnitude $\sqrt{10}$.

Statement II : In a triangle ABC , $\cos 2A + \cos 2B + \cos 2C \geq -\frac{3}{2}$.

- (A) Both Statement I and Statement II are correct.
 (B) Both Statement I and Statement II are incorrect.
 (C) Statement I is correct but Statement II is incorrect.
 (D) Statement I is incorrect but Statement II is correct.

Q121. JEE Main 2024

Let $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = 11\hat{i} - \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $(\vec{a} + \vec{b}) \times \vec{c} = \vec{c} \times (-2\vec{a} + 3\vec{b})$.

If $(2\vec{a} + 3\vec{b}) \cdot \vec{c} = 1670$, then $|\vec{c}|^2$ is equal to:

- (A) 1600
 (B) 1618
 (C) 1627
 (D) 1609

Q122. JEE Main 2024

Let $\vec{a} = 9\hat{i} - 13\hat{j} + 25\hat{k}$, $\vec{b} = 3\hat{i} + 7\hat{j} - 13\hat{k}$ and $\vec{c} = 17\hat{i} - 2\hat{j} + \hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} = (\vec{b} + \vec{c}) \times \vec{a}$ and $\vec{r} \cdot (\vec{b} - \vec{c}) = 0$, then $\frac{1593\vec{r} + 67\vec{a}}{(593)^2}$ is equal to _____.

Q123. JEE Main 2024

Let $\vec{a} = \hat{i} - 3\hat{j} + 7\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $(\vec{a} + 2\vec{b}) \times \vec{c} = 3(\vec{c} \times \vec{a})$.

If $\vec{a} \cdot \vec{c} = 130$, then $\vec{b} \cdot \vec{c}$ is equal to _____.

Q124. JEE Main 2024

Let $\vec{a} = 2\hat{i} + 5\hat{j} - \hat{k}$, $\vec{b} = 2\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{c}$ be three vectors such that $(\vec{c} + \hat{i}) \times (\vec{a} + \vec{b} + \hat{i}) = \vec{a} \times (\vec{c} + \hat{i})$. If $\vec{a} \cdot \vec{c} = -29$, then $\vec{c} \cdot (-2\hat{i} + \hat{j} + \hat{k})$ is equal to:

- (A) 15 (B) 10 (C) 5 (D) 12

Q125. JEE Main 2023

Let $\vec{a} = \hat{i} + 2\hat{j} + \lambda\hat{k}$, $\vec{b} = 3\hat{i} - 5\hat{j} - \lambda\hat{k}$, $\vec{a} \cdot \vec{c} = 7$, $2\vec{b} \cdot \vec{c} + 43 = 0$, $\vec{a} \times \vec{c} = \vec{b} \times \vec{c}$. Then $|\vec{a} \cdot \vec{b}|$ is equal to :

Q126. JEE Main 2023

Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that

$$|\vec{a}| = \sqrt{31}, 4|\vec{b}| = |\vec{c}| = 2 \text{ and } 2(\vec{a} \times \vec{b}) = 3(\vec{c} \times \vec{a}).$$

If the angle between $\vec{b}$ and $\vec{c}$ is $\frac{2\pi}{3}$, then $(\frac{\vec{a} \times \vec{c}}{\vec{a} \cdot \vec{b}})^2$ is equal to _____.

Q127. JEE Main 2026

Let $\vec{a} = 2\hat{i} - 5\hat{j} + 5\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + 3\hat{k}$. If $\vec{c}$ is a vector such that $2(\vec{a} \times \vec{c}) + 3(\vec{b} \times \vec{c}) = \vec{0}$ and $(\vec{a} - \vec{b}) \cdot \vec{c} = -97$, then $|\vec{c} \times \hat{k}|^2$ is equal to

- (A) 193
 (B) 205
 (C) 218
 (D) 233

Q128. JEE Main 2026

Let $\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}$, $\vec{b} = 10\hat{i} + 2\hat{j} - \hat{k}$ and a vector $\vec{c}$ be such that $2(\vec{a} \times \vec{b}) + 3(\vec{b} \times \vec{c}) = \vec{0}$.

If $\vec{a} \cdot \vec{c} = 15$, then $\vec{c} \cdot (\hat{i} + \hat{j} - 3\hat{k})$ is equal to :

- (A) -6
 (B) -5
 (C) -4
 (D) -3

Q129. JEE Main 2025

Let $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} - 5\hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} \times \vec{c} = \vec{a} \times \vec{b} = \vec{c} \times \vec{b}$ and $(\vec{a} + \vec{c}) \cdot (\vec{b} + \vec{c}) = 168$. Then the maximum value of $|\vec{c}|^2$ is :

- (A) 77
 (B) 154
 (C) 308
 (D) 462

Q130. JEE Main 2023

Let $\lambda \in \mathbb{Z}$, $\vec{a} = \lambda\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$. Let $\vec{c}$ be a vector such that $(\vec{a} + \vec{b} + \vec{c}) \times \vec{c} = \vec{0}$, $\vec{a} \cdot \vec{c} = -17$ and $\vec{b} \cdot \vec{c} = -20$. Then $|\vec{c} \times (\lambda\hat{i} + \hat{j} + \hat{k})|^2$ is equal to :

- (A) 53
 (B) 62
 (C) 49
 (D) 46

Q131. JEE Main 2026

Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $\vec{a} \times \vec{b} = 2(\vec{a} \times \vec{c})$. If $|\vec{a}| = 1$, $|\vec{b}| = 4$, $|\vec{c}| = 2$, and the angle between $\vec{b}$ and $\vec{c}$ is 60° , then $|\vec{a} \cdot \vec{c}|$ is equal to

- (A) 2 (B) 0
 (C) 1 (D) 4

