

**Topic: Vector Algebra**

Mathematics

Topic-wise JEE Main PYQ Part-2

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Topic 1: Scalar Triple Product**Q1. JEE Main 2024**

Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{b}| = 1$ and $|\vec{b} \times \vec{a}| = 2$. Then $|\vec{b} \times \vec{a} - \vec{b}|^2$ is equal to

- (A) 1 (B) 3 (C) 5 (D) 4

Q2. JEE Main 2024

Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ be two vectors such that $|\vec{a}| = 1$, $\vec{a} \cdot \vec{b} = 2$ and $|\vec{b}| = 4$. If $\vec{c} = 2(\vec{a} \times \vec{b}) - 3\vec{b}$, then the angle between $\vec{b}$ and $\vec{c}$ is equal to:

- (A) $\cos^{-1}\left(-\frac{1}{\sqrt{3}}\right)$
 (B) $\cos^{-1}\left(\frac{2}{3}\right)$
 (C) $\cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$
 (D) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

Q3. JEE Main 2024

Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}| = 1$, $|\vec{b}| = 4$, and $\vec{a} \cdot \vec{b} = 2$. If $\vec{c} = (2\vec{a} \times \vec{b}) - 3\vec{b}$ and the angle between $\vec{b}$ and $\vec{c}$ is α , then $192 \sin^2 \alpha$ is equal to _____.

Q4. JEE Main 2024

Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 3(\hat{i} - \hat{j} + \hat{k})$. Let $\vec{c}$ be the vector such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$. Then $\vec{a} \cdot ((\vec{c} \times \vec{b}) - \vec{b} - \vec{c})$ is equal to:

- (A) 32 (B) 36 (C) 24 (D) 20

Q5. JEE Main 2024

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = x\hat{i} + 2\hat{j} + 3\hat{k}$, $x \in \mathbb{R}$.

If $\vec{d}$ is the unit vector in the direction of $\vec{b} + \vec{c}$ such that $\vec{a} \cdot \vec{d} = 1$, then $(\vec{a} \times \vec{b}) \cdot \vec{c}$ is equal to

- (A) 3 (B) 9
(C) 11 (D) 6

Q6. JEE Main 2023

Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$. If $\vec{c}$ is a vector such that $\vec{a} \cdot \vec{c} = 11$, $\vec{b} \cdot (\vec{a} \times \vec{c}) = 27$ and $\vec{b} \cdot \vec{c} = -\sqrt{3}|\vec{b}|$, then $|\vec{a} \times \vec{c}|^2$ is equal to _____.

Q7. JEE Main 2023

Let $\vec{a} = 4\hat{i} + 3\hat{j}$ and $\vec{b} = 3\hat{i} - 4\hat{j} + 5\hat{k}$. If $\vec{c}$ is a vector such that $\vec{c} \cdot (\vec{a} \times \vec{b}) + 25 = 0$, $\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = 4$, and projection of $\vec{c}$ on $\vec{a}$ is 1, then the projection of $\vec{c}$ on $\vec{b}$ equals:

- (A) $\frac{3}{\sqrt{2}}$
 (B) $\frac{1}{\sqrt{2}}$
 (C) $\frac{1}{5}$
 (D) $\frac{5}{\sqrt{2}}$

Q8. JEE Main 2021

Let $\vec{a} = \hat{i} - \alpha\hat{j} + \beta\hat{k}$, $\vec{b} = 3\hat{i} + \beta\hat{j} - \alpha\hat{k}$ and $\vec{c} = -\alpha\hat{i} - 2\hat{j} + \hat{k}$, where α and β are integers. If $\vec{a} \cdot \vec{b} = -1$ and $\vec{b} \cdot \vec{c} = 10$, then $(\vec{a} \times \vec{b}) \cdot \vec{c}$ is equal to _____.

Q9. JEE Main 2021

If $\vec{a} = \alpha\hat{i} + \beta\hat{j} + 3\hat{k}$, $\vec{b} = -\beta\hat{i} - \alpha\hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} - \hat{k}$ such that $\vec{a} \cdot \vec{b} = 1$ and $\vec{b} \cdot \vec{c} = -3$, then $\frac{1}{3}((\vec{a} \times \vec{b}) \cdot \vec{c})$ is equal to _____.

Q10. JEE Main 2014

If $[\vec{a} \times \vec{b} \quad \vec{b} \times \vec{c} \quad \vec{c} \times \vec{a}] = \lambda [\vec{a} \quad \vec{b} \quad \vec{c}]^2$ then λ is equal to:

- (A) 0 (B) 1 (C) 2 (D) 3

Q11. JEE Main 2009

If $\vec{u}, \vec{v}, \vec{w}$ are non-coplanar vectors and p, q are real numbers, then the equality $[3\vec{u} \quad p\vec{v} \quad p\vec{w}] - [p\vec{v} \quad \vec{w} \quad q\vec{u}] - [2\vec{w} \quad q\vec{v} \quad q\vec{u}] = 0$ holds for:

- (A) exactly two values of (p, q)
 (B) more than two but not all values of (p, q)
 (C) all values of (p, q)
 (D) exactly one value of (p, q)

Q12. JEE Main 2005

If $\vec{a}, \vec{b}, \vec{c}$ are non coplanar vectors and λ is a real number then $[\lambda(\vec{a} + \vec{b}) \quad \lambda^2\vec{b} \quad \lambda\vec{c}] = [\vec{a} \quad \vec{b} + \vec{c} \quad \vec{b}]$ for:

- (A) exactly one value of λ
 (B) no value of λ
 (C) exactly three values of λ
 (D) exactly two values of λ

Q13. JEE Main 2003

If $\vec{u}, \vec{v}$ and $\vec{w}$ are three non-coplanar vectors, then $(\vec{u} + \vec{v} - \vec{w}) \cdot (\vec{u} - \vec{v}) \times (\vec{v} - \vec{w})$ equals :

- (A) $3\vec{u} \cdot \vec{v} \times \vec{w}$
 (B) 0
 (C) $\vec{u} \cdot \vec{v} \times \vec{w}$
 (D) $\vec{u} \cdot \vec{w} \times \vec{v}$

Q14. JEE Main 2021

Let a vector $\vec{a}$ be coplanar with vectors $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + \hat{k}$. If $\vec{a}$ is perpendicular to $\vec{d} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, and $|\vec{a}| = \sqrt{10}$. Then a possible value of $[\vec{a} \ \vec{b} \ \vec{c}] + [\vec{a} \ \vec{b} \ \vec{d}] + [\vec{a} \ \vec{c} \ \vec{d}]$ is equal to :

- (A) -42 (B) -40 (C) -29 (D) -38

Topic 2: Coplanar

Q15. JEE Main 2023

Let a, b, c be three distinct real numbers, none equal to one. If the vectors $a\hat{i} + \hat{j} + \hat{k}, \hat{i} + b\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + c\hat{k}$ are coplanar, then $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$ is equal to :

- (A) -2 (B) 1 (C) -1 (D) 2

Q16. JEE Main 2023

If four distinct points with position vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are coplanar, then $[\vec{a} \ \vec{b} \ \vec{c}]$ is equal to :

- (A) $[\vec{d} \ \vec{b} \ \vec{a}] + [\vec{a} \ \vec{c} \ \vec{d}] + [\vec{d} \ \vec{b} \ \vec{c}]$
 (B) $[\vec{b} \ \vec{c} \ \vec{d}] + [\vec{d} \ \vec{a} \ \vec{c}] + [\vec{d} \ \vec{b} \ \vec{a}]$
 (C) $[\vec{a} \ \vec{d} \ \vec{b}] + [\vec{d} \ \vec{c} \ \vec{a}] + [\vec{d} \ \vec{b} \ \vec{c}]$
 (D) $[\vec{d} \ \vec{c} \ \vec{a}] + [\vec{b} \ \vec{d} \ \vec{a}] + [\vec{c} \ \vec{d} \ \vec{b}]$

Q17. JEE Main 2023

Let S be the set of all (λ, μ) for which the vectors $\lambda\hat{i} - \hat{j} + \hat{k}, \hat{i} + 2\hat{j} + \mu\hat{k}$ and $3\hat{i} - 4\hat{j} + 5\hat{k}$, where $\lambda - \mu = 5$, are coplanar, then $\sum_{(\lambda, \mu) \in S} 80(\lambda^2 + \mu^2)$ is equal to :

- (A) 2370 (B) 2130 (C) 2210 (D) 2290

Q18. JEE Main 2023

If the vectors $\vec{a} = \lambda\hat{i} + \mu\hat{j} + 4\hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} + 3\hat{j} + \hat{k}$ are coplanar and the projection of $\vec{a}$ on the vector $\vec{b}$ is $\sqrt{54}$ units, then the sum of all possible values of $\lambda + \mu$ is equal to :

- (A) 24 (B) 0 (C) 18 (D) 6

Q19. JEE Main 2023

Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero non-coplanar vectors. Let the position vectors of four points A, B, C and D be $\vec{a} - \vec{b} + \vec{c}, \lambda\vec{a} - 3\vec{b} + 4\vec{c}, -\vec{a} + 2\vec{b} - 3\vec{c}$ and $2\vec{a} - 4\vec{b} + 6\vec{c}$ respectively. If $\vec{AB}, \vec{AC}$ and $\vec{AD}$ are coplanar, then λ is equal to _____.

Q20. JEE Main 2023

If the four points, whose position vectors are $3\hat{i} - 4\hat{j} + 2\hat{k}, \hat{i} + 2\hat{j} - \hat{k}, -2\hat{i} - \hat{j} + 3\hat{k}$ and $5\hat{i} - 2\alpha\hat{j} + 4\hat{k}$ are coplanar, then α is equal to :

- (A) $\frac{73}{17}$ (B) $-\frac{73}{17}$ (C) $-\frac{107}{17}$ (D) $\frac{107}{17}$

Q21. JEE Main 2023

Let the vectors $\vec{u}_1 = \hat{i} + \hat{j} + a\hat{k}, \vec{u}_2 = \hat{i} + b\hat{j} + \hat{k}$ and $\vec{u}_3 = c\hat{i} + \hat{j} + \hat{k}$ be coplanar. If the vectors $\vec{v}_1 = (a+b)\hat{i} + c\hat{j} + c\hat{k}, \vec{v}_2 = a\hat{i} + (b+c)\hat{j} + a\hat{k}$ and $\vec{v}_3 = b\hat{i} + b\hat{j} + (c+a)\hat{k}$ are also coplanar, then $6(a+b+c)$ is equal to :

- (A) 12 (B) 6 (C) 0 (D) 4

Q22. JEE Main 2023

Let the position vectors of the points A, B, C and D be $5\hat{i} + 5\hat{j} + 2\lambda\hat{k}, \hat{i} + 2\hat{j} + 3\hat{k}, -2\hat{i} + \lambda\hat{j} + 4\hat{k}$ and $-\hat{i} + 5\hat{j} + 6\hat{k}$.

Let the set $S = \{\lambda \in \mathbb{R} :$

the points A, B, C and D are coplanar }.

Then $\sum_{\lambda \in S} (\lambda + 2)^2$ is equal to :

- (A) $\frac{37}{2}$ (B) 25 (C) 13 (D) 41

Q23. JEE Main 2023

The sum of all values of α , for which the points whose position vectors are $\hat{i} - 2\hat{j} + 3\hat{k}, 2\hat{i} - 3\hat{j} + 4\hat{k}, (\alpha + 1)\hat{i} + 2\hat{k}$ and $9\hat{i} + (\alpha - 8)\hat{j} + 6\hat{k}$ are coplanar, is equal to :

- (A) 6 (B) 4 (C) -2 (D) 2

Q24. JEE Main 2022

If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}, \vec{b} = 3\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ are coplanar vectors and $\vec{a} \cdot \vec{c} = 5, \vec{b} \perp \vec{c}$, then $122(c_1 + c_2 + c_3)$ is equal to _____.

Q25. JEE Main 2021

Let the vectors

$(2+a+b)\hat{i} + (a+2b+c)\hat{j} - (b+c)\hat{k}, (1+b)\hat{i} + 2b\hat{j} - b\hat{k}$

and $(2+b)\hat{i} + 2b\hat{j} + (1-b)\hat{k}, a, b, c, \in \mathbb{R}$ be co-planar. Then which of the following is true?

- (A) $2b = a + c$ (B) $3c = a + b$
 (C) $a = b + 2c$ (D) $2a = b + c$

Q26. JEE Main 2021

Let a, b and c be distinct positive numbers. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}, \hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ are co-planar, then c is equal to :

(A) $\frac{2}{a+b}$
(C) $\frac{1}{a} + \frac{1}{b}$

(B) $\frac{a+b}{2}$
(D) $\sqrt{ab}$

Q27. JEE Main 2020

If the vectors, $\vec{p} = (a+1)\hat{i} + a\hat{j} + a\hat{k}$,
 $\vec{q} = a\hat{i} + (a+1)\hat{j} + a\hat{k}$ and
 $\vec{r} = a\hat{i} + a\hat{j} + (a+1)\hat{k}$ ($a \in \mathbb{R}$) are coplanar
and $3(\vec{p} \cdot \vec{q})^2 - \lambda |\vec{r} \times \vec{q}|^2 = 0$, then the value of λ is _____.

Q28. JEE Main 2019

Let $\vec{a} = \hat{i} + 2\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} + \lambda\hat{j} + 4\hat{k}$ and $\vec{c} = 2\hat{i} + 4\hat{j} + (\lambda^2 - 1)\hat{k}$ be coplanar vectors. Then the non-zero vector $\vec{a} \times \vec{c}$ is :

- (A) $-10\hat{i} - 5\hat{j}$
(B) $-10\hat{i} + 5\hat{j}$
(C) $-14\hat{i} + 5\hat{j}$
(D) $-14\hat{i} - 5\hat{j}$

Q29. JEE Main 2019

Let $\alpha \in \mathbb{R}$ and the three vectors
 $\vec{a} = \alpha\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \alpha\hat{k}$ and $\vec{c} = \alpha\hat{i} - 2\hat{j} + 3\hat{k}$.
Then the set
 $S = \{\alpha : \vec{a}, \vec{b} \text{ and } \vec{c} \text{ are coplanar}\}$:

- (A) contains exactly two numbers only one of which is positive
(B) is singleton
(C) contains exactly two positive numbers
(D) is empty

Q30. JEE Main 2019

The sum of the distinct real values of μ , for which the vectors,
 $\mu\hat{i} + \hat{j} + \hat{k}$, $\hat{i} + \mu\hat{j} + \hat{k}$, $\hat{i} + \hat{j} + \mu\hat{k}$ are co-planar, is :

- (A) 2 (B) -1
(C) 0 (D) 1

Q31. JEE Main 2007

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$. If the vectors $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then x equals :

- (A) -4 (B) -2 (C) 0 (D) 1.

Q32. JEE Main 2004

If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and λ is a real number, then the vectors $\vec{a} + 2\vec{b} + 3\vec{c}$, $\lambda\vec{b} + 4\vec{c}$ and $(2\lambda - 1)\vec{c}$ are non coplanar for :

- (A) no value of λ
(B) all except one value of λ
(C) all except two values of λ
(D) all values of λ

Q33. JEE Main 2021

Let O be the origin. Let $\vec{OP} = x\hat{i} + y\hat{j} - \hat{k}$ and $\vec{OQ} = -\hat{i} + 2\hat{j} + 3x\hat{k}$, $x, y \in \mathbb{R}$, $x > 0$, be such that $|\vec{PQ}| = \sqrt{20}$ and the vector $\vec{OP}$ is perpendicular $\vec{OQ}$. If $\vec{OR} = 3\hat{i} + z\hat{j} - 7\hat{k}$, $z \in \mathbb{R}$, is coplanar with $\vec{OP}$ and $\vec{OQ}$, then the value of $x^2 + y^2 + z^2$ is equal to :

- (A) 2 (B) 9 (C) 7 (D) 1

Topic 3: Application of STP Volumn

Q34. JEE Main 2020

Let the volume of a parallelopiped whose coterminous edges are given by
 $\vec{u} = \hat{i} + \hat{j} + \lambda\hat{k}$, $\vec{v} = \hat{i} + \hat{j} + 3\hat{k}$ and
 $\vec{w} = 2\hat{i} + \hat{j} + \hat{k}$ be 1 cu. unit. If θ be the angle between the edges $\vec{u}$ and $\vec{w}$, then $\cos\theta$ can be :

- (A) $\frac{7}{6\sqrt{3}}$ (B) $\frac{7}{6\sqrt{6}}$ (C) $\frac{5}{7}$ (D) $\frac{5}{3\sqrt{3}}$

Q35. JEE Main 2020

If the volume of a parallelopiped, whose coterminous edges are given by the vectors $\vec{a} = \hat{i} + \hat{j} + n\hat{k}$,
 $\vec{b} = 2\hat{i} + 4\hat{j} - n\hat{k}$ and
 $\vec{c} = \hat{i} + n\hat{j} + 3\hat{k}$ ($n \geq 0$), is 158 cu. units, then :

- (A) $n = 7$ (B) $\vec{b} \cdot \vec{c} = 10$
(C) $\vec{a} \cdot \vec{c} = 17$ (D) $n = 9$

Q36. JEE Main 2019

If the volume of parallelopiped formed by the vectors $\hat{i} + \lambda\hat{j} + \hat{k}$, $\hat{j} + \lambda\hat{k}$ and $\lambda\hat{i} + \hat{k}$ is minimum, then λ is equal to :

- (A) $-\frac{1}{\sqrt{3}}$ (B) $\sqrt{3}$ (C) $-\sqrt{3}$ (D) $\frac{1}{\sqrt{3}}$

Q37. JEE Main 2023

Let the vectors $\vec{a}, \vec{b}, \vec{c}$ represent three coterminous edges of a parallelopiped of volume V. Then the volume of the parallelopiped, whose coterminous edges are represented by $\vec{a}, \vec{b} + \vec{c}$ and $\vec{a} + 2\vec{b} + 3\vec{c}$ is equal to :

- (A) 3 V (B) 2 V (C) 6 V (D) V

Topic 4: STP-2

Q38. JEE Main 2025

Let $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} + 2\hat{j} + 5\hat{k}$ and a vector $\vec{c}$ be such that $(\vec{a} - \vec{c}) \times \vec{b} = -18\hat{i} - 3\hat{j} + 12\hat{k}$ and $\vec{a} \cdot \vec{c} = 3$. If $\vec{b} \times \vec{c} = \vec{d}$, then $|\vec{a} \cdot \vec{d}|$ is equal to :

- (A) 15 (B) 18 (C) 12 (D) 9

Q39. JEE Main 2025

Let the angle θ , $0 < \theta < \frac{\pi}{2}$ between two unit vectors $\hat{a}$ and $\hat{b}$ be $\sin^{-1}\left(\frac{\sqrt{65}}{9}\right)$. If the vector $\vec{c} = 3\hat{a} + 6\hat{b} + 9(\hat{a} \times \hat{b})$, then the value of $9(\vec{c} \cdot \hat{a}) - 3(\vec{c} \cdot \hat{b})$ is

(A) 31 (B) 29 (C) 24 (D) 27

Q40. JEE Main 2023

Let $\vec{v} = \alpha\hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{w} = 2\alpha\hat{i} + \hat{j} - \hat{k}$ and $\vec{u}$ be a vector such that $|\vec{u}| = \alpha > 0$. If the minimum value of the scalar triple product $[\vec{u} \ \vec{v} \ \vec{w}]$ is $-\alpha\sqrt{3401}$, and $|\vec{u} \cdot \hat{i}|^2 = \frac{m}{n}$ where m and n are coprime natural numbers, then $m + n$ is equal to _____.

Q41. JEE Main 2022

Let a vector $\vec{c}$ be coplanar with the vectors $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$. If the vector $\vec{c}$ also satisfies the conditions $\vec{c} \cdot [(\vec{a} + \vec{b}) \times (\vec{a} \times \vec{b})] = -42$ and $(\vec{c} \times (\vec{a} - \vec{b})) \cdot \hat{k} = 3$, then the value of $|\vec{c}|^2$ is equal to :

- (A) 24 (B) 29
(C) 35 (D) 42

Q42. JEE Main 2022

Let the vectors $\vec{a} = (1+t)\hat{i} + (1-t)\hat{j} + \hat{k}$, $\vec{b} = (1-t)\hat{i} + (1+t)\hat{j} + 2\hat{k}$ and $\vec{c} = t\hat{i} - t\hat{j} + \hat{k}$, $t \in \mathbf{R}$ be such that for $\alpha, \beta, \gamma \in \mathbf{R}$, $\alpha\vec{a} + \beta\vec{b} + \gamma\vec{c} = \vec{0} \Rightarrow \alpha = \beta = \gamma = 0$. Then, the set of all values of t is :

- (A) a non-empty finite set
(B) equal to $\mathbf{N}$
(C) equal to $\mathbf{R} - \{0\}$
(D) equal to $\mathbf{R}$

Q43. JEE Main 2021

Let three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ be such that $\vec{a} \times \vec{b} = \vec{c}$, $\vec{b} \times \vec{c} = \vec{a}$ and $|\vec{a}| = 2$. Then which one of the following is not true?

- (A) $\vec{a} \times ((\vec{b} + \vec{c}) \times (\vec{b} \times \vec{c})) = \vec{0}$
(B) Projection of $\vec{a}$ on $(\vec{b} \times \vec{c})$ is 2
(C) $[\vec{a} \ \vec{b} \ \vec{c}] + [\vec{c} \ \vec{a} \ \vec{b}] = 8$
(D) $|3\vec{a} + \vec{b} - 2\vec{c}|^2 = 51$

Q44. JEE Main 2021

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$. If $\vec{c}$ is a vector such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$, then $\vec{a} \cdot (\vec{b} \times \vec{c})$ is equal to :

- (A) -2 (B) -6 (C) 6 (D) 2

Q45. JEE Main 2020

Let x_0 be the point of Local maxima of $f(x) = \vec{a} \cdot (\vec{b} \times \vec{c})$, where

$$\vec{a} = x\hat{i} - 2\hat{j} + 3\hat{k}, \quad \vec{b} = -2\hat{i} + x\hat{j} - \hat{k}, \quad \vec{c} = 7\hat{i} - 2\hat{j} + x\hat{k}.$$

Then the value of

$$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} \text{ at } x = x_0 \text{ is :}$$

- (A) 14 (B) -30 (C) -4 (D) -22

Topic 5: VTP-1

Q46. JEE Main 2026

Let $\vec{a} = -\hat{i} + \hat{j} + 2\hat{k}$, $\vec{b} = \hat{i} - \hat{j} - 3\hat{k}$, $\vec{c} = \vec{a} \times \vec{b}$ and $\vec{d} = \vec{c} \times \vec{a}$. Then $(\vec{a} - \vec{b}) \cdot \vec{d}$ is equal to :

- (A) 2
(B) -2
(C) 4
(D) -4

Q47. JEE Main 2024

Let $\vec{a} = -5\hat{i} + \hat{j} - 3\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 4\hat{k}$ and $\vec{c} = (((\vec{a} \times \vec{b}) \times \hat{i}) \times \hat{i}) \times \hat{i}$. Then $\vec{c} \cdot (-\hat{i} + \hat{j} + \hat{k})$ is equal to :

- (A) -12 (B) -10 (C) -13 (D) -15

Q48. JEE Main 2021

If $\vec{a}$ and $\vec{b}$ are perpendicular, then $\vec{a} \times (\vec{a} \times (\vec{a} \times (\vec{a} \times \vec{b})))$ is equal to :

- (A) $\frac{1}{2}|\vec{a}|^4\vec{b}$ (B) $\vec{0}$
(C) $\vec{a} \times \vec{b}$ (D) $|\vec{a}|^4\vec{b}$

Q49. JEE Main 2021

Let $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + 2\hat{j} + 3\hat{k}$. Then the vector product $(\vec{a} + \vec{b}) \times ((\vec{a} \times ((\vec{a} - \vec{b}) \times \vec{b})) \times \vec{b})$ is equal to :

- (A) $5(34\hat{i} - 5\hat{j} + 3\hat{k})$
(B) $7(34\hat{i} - 5\hat{j} + 3\hat{k})$
(C) $7(30\hat{i} - 5\hat{j} + 7\hat{k})$
(D) $5(30\hat{i} - 5\hat{j} + 7\hat{k})$

Q50. JEE Main 2022

Let $\vec{a} = 2\hat{i} - \hat{j} + 5\hat{k}$ and $\vec{b} = \alpha\hat{i} + \beta\hat{j} + 2\hat{k}$. If $((\vec{a} \times \vec{b}) \times \hat{i}) \cdot \hat{k} = \frac{23}{2}$, then $|\vec{b} \times 2\hat{j}|$ is equal to :

- (A) 4 (B) 5 (C) $\sqrt{21}$ (D) $\sqrt{17}$

Q51. JEE Main 2020

If $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$, then the value of $|\hat{i} \times (\vec{a} \times \hat{i})|^2 + |\hat{j} \times (\vec{a} \times \hat{j})|^2 + |\hat{k} \times (\vec{a} \times \hat{k})|^2$ is equal to _____

Q52. JEE Main 2023

Let $\lambda \in \mathbf{R}$, $\vec{a} = \lambda\hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{b} = \hat{i} - \lambda\hat{j} + 2\hat{k}$. If $((\vec{a} + \vec{b}) \times (\vec{a} \times \vec{b})) \times (\vec{a} - \vec{b}) = 8\hat{i} - 40\hat{j} - 24\hat{k}$, then $|\lambda(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})|^2$ is equal to :

- (A) 136
(B) 140
(C) 144
(D) 132

Topic 6: VTP-2

Q53. JEE Main 2025

Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} + \hat{j} - \hat{k}$ and $\vec{c}$ be three vectors such that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$. If the vector $\vec{C}$ is perpendicular to $\vec{b}$ and $\vec{a} \cdot \vec{c} = 5$, then $|\vec{c}|$ is equal to

- (A) $\sqrt{\frac{11}{6}}$ (B) $\frac{1}{3\sqrt{2}}$
(C) 18 (D) 16

Q54. JEE Main 2021

Let three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ be such that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$,
 $\vec{a} \cdot \vec{c} = 7$ and

$\vec{b}$ is perpendicular to $\vec{c}$, where
 $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{k}$, then the value of $2|\vec{a} + \vec{b} + \vec{c}|^2$ is ____.

Q55. JEE Main 2023

If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non-zero vectors and $\hat{n}$ is a unit vector perpendicular to $\vec{c}$ such that $\vec{a} = \alpha \vec{b} - \hat{n}$, ($\alpha \neq 0$) and $\vec{b} \cdot \vec{c} = 12$, then $|\vec{c} \times (\vec{a} \times \vec{b})|$ is equal to :

- (A) 15 (B) 9 (C) 6 (D) 12

Q56. JEE Main 2022

Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three coplanar concurrent vectors such that angles between any two of them is same. If the product of their magnitudes is 14 and
 $(\vec{a} \times \vec{b}) \cdot (\vec{b} \times \vec{c}) + (\vec{b} \times \vec{c}) \cdot (\vec{c} \times \vec{a}) + (\vec{c} \times \vec{a}) \cdot (\vec{a} \times \vec{b}) = 168$,
then $|\vec{a}| + |\vec{b}| + |\vec{c}|$ is equal to :

- (A) 10 (B) 14 (C) 16 (D) 18

Q57. JEE Main 2021

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a} = \vec{b} \times (\vec{b} \times \vec{c})$. If magnitudes of the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are $\sqrt{2}$, 1 and 2 respectively and the angle between $\vec{b}$ and $\vec{c}$ is θ ($0 < \theta < \frac{\pi}{2}$), then the value of $1 + \tan\theta$ is equal to :

- (A) $\sqrt{3} + 1$ (B) 2 (C) 1 (D) $\frac{\sqrt{3}+1}{\sqrt{3}}$

Q58. JEE Main 2019

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three unit vectors, out of which vectors $\vec{b}$ and $\vec{c}$ are non-parallel. If α and β are the angles which vector $\vec{a}$ makes with vectors $\vec{b}$ and $\vec{c}$ respectively and $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{1}{2} \vec{b}$, then $|\alpha - \beta|$ is equal to :

- (A) 90° (B) 30° (C) 45° (D) 60°

Q59. JEE Main 2016

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2} (\vec{b} + \vec{c})$. If $\vec{b}$ is not parallel to $\vec{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is:

- (A) $\frac{2\pi}{3}$ (B) $\frac{5\pi}{6}$ (C) $\frac{3\pi}{4}$ (D) $\frac{\pi}{2}$

Q60. JEE Main 2022

Let $\vec{a} = 3\hat{i} + \hat{j}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$. Let $\vec{c}$ be a vector satisfying $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b} + \lambda \vec{c}$. If $\vec{b}$ and $\vec{c}$ are non-parallel, then the value of λ is :

- (A) -5 (B) 5
(C) 1 (D) -1

Q61. JEE Main 2015

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of them are collinear and
 $(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3} |\vec{b}| |\vec{c}| \vec{a}$. If θ is the angle between vectors $\vec{b}$ and $\vec{c}$, then a value of $\sin \theta$ is :

- (A) $\frac{2}{3}$ (B) $\frac{-2\sqrt{3}}{3}$ (C) $\frac{2\sqrt{2}}{3}$ (D) $\frac{-\sqrt{2}}{3}$

Q62. JEE Main 2022

If $\vec{a} \cdot \vec{b} = 1$, $\vec{b} \cdot \vec{c} = 2$ and $\vec{c} \cdot \vec{a} = 3$, then the value of $[\vec{a} \times (\vec{b} \times \vec{c}), \vec{b} \times (\vec{c} \times \vec{a}), \vec{c} \times (\vec{b} \times \vec{a})]$ is :

- (A) 0
(B) $-6\vec{a} \cdot (\vec{b} \times \vec{c})$
(C) $-12\vec{c} \cdot (\vec{a} \times \vec{b})$
(D) $-12\vec{b} \cdot (\vec{c} \times \vec{a})$

Q63. JEE Main 2004

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be non-zero vectors such that $(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3} |\vec{b}| |\vec{c}| \vec{a}$. If θ is the acute angle between the vectors $\vec{b}$ and $\vec{c}$, then $\sin\theta$ equals :

- (A) $\frac{2\sqrt{2}}{3}$ (B) $\frac{\sqrt{2}}{3}$ (C) $\frac{2}{3}$ (D) $\frac{1}{3}$

Q64. JEE Main 2002

If $\vec{a}$, $\vec{b}$, $\vec{c}$ are vectors such that $[\vec{a} \vec{b} \vec{c}] = 4$ then $[\vec{a} \times \vec{b} \vec{b} \times \vec{c} \vec{c} \times \vec{a}] =$

- (A) 16 (B) 64 (C) 4 (D) 8

Q65. JEE Main 2011

If $\vec{a} = \frac{1}{\sqrt{10}} (3\hat{i} + \hat{k})$ and $\vec{b} = \frac{1}{7} (2\hat{i} + 3\hat{j} - 6\hat{k})$, then the value of $(2\vec{a} - \vec{b}) [(\vec{a} \times \vec{b}) \times (\vec{a} + 2\vec{b})]$ is :

- (A) -3 (B) 5 (C) 3 (D) -5

Q66. JEE Main 2006

If $(\vec{a} \times \vec{b}) \times \vec{c} = \vec{a} \times (\vec{b} \times \vec{c})$ where $\vec{a}$, $\vec{b}$ and $\vec{c}$ are any three vectors such that $\vec{a} \cdot \vec{b} \neq 0$, $\vec{b} \cdot \vec{c} \neq 0$ then $\vec{a}$ and $\vec{c}$ are :

- (A) inclined at an angle of $\frac{\pi}{3}$ between them
(B) inclined at an angle of $\frac{\pi}{6}$ between them
(C) perpendicular
(D) parallel

Q67. JEE Main 2024

Let $\vec{a} = 6\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $|\vec{c}| \geq 6$, $\vec{a} \cdot \vec{c} = 6|\vec{c}|$, $|\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $\vec{a} \times \vec{b}$ and $\vec{c}$ is 60° , then $|(\vec{a} \times \vec{b}) \times \vec{c}|$ is equal to:

- (A) $\frac{3}{2}\sqrt{6}$ (B) $\frac{9}{2}(6 - \sqrt{6})$
(C) $\frac{9}{2}(6 + \sqrt{6})$ (D) $\frac{3}{2}\sqrt{3}$

Q68. JEE Main 2021

Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $\vec{a} \cdot \vec{c} = |\vec{c}|$, $|\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $(\vec{a} \times \vec{b})$ and $\vec{c}$ is $\frac{\pi}{6}$, then the value of $|(\vec{a} \times \vec{b}) \times \vec{c}|$ is:

- (A) $\frac{2}{3}$ (B) 4
(C) 3 (D) $\frac{3}{2}$

Q69. JEE Main 2020

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 5$, $\vec{b} \cdot \vec{c} = 10$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$. If $\vec{a}$ is perpendicular to the vector $\vec{b} \times \vec{c}$, then $|\vec{a} \times (\vec{b} \times \vec{c})|$ is equal to ____.

Q70. JEE Main 2017

Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. Let $\vec{c}$ be a vector such that $|\vec{c} - \vec{a}| = 3$, $|(\vec{a} \times \vec{b}) \times \vec{c}| = 3$ and the angle between $\vec{c}$ and $\vec{a} \times \vec{b}$ is 30° . Then $\vec{a} \cdot \vec{c}$ is equal to:

- (A) 2 (B) 5 (C) $\frac{1}{8}$ (D) $\frac{25}{8}$

Topic 7: Scalar Triple Product of 4 Vectors**Q71. JEE Main 2023**

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non zero vectors such that $\vec{b} \cdot \vec{c} = 0$ and $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\vec{b} - \vec{c}}{2}$. If $\vec{d}$ be a vector such that $\vec{b} \cdot \vec{d} = \vec{a} \cdot \vec{b}$, then $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d})$ is equal to

- (A) $\frac{1}{2}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{4}$ (D) $\frac{3}{4}$

Topic 8: Angle Bisector**Q72. JEE Main 2026**

Let P be a point in the plane of the vectors $\vec{AB} = 3\hat{i} + \hat{j} - \hat{k}$ and $\vec{AC} = \hat{i} - \hat{j} + 3\hat{k}$ such that P is equidistant from the lines AB and AC. If $|\vec{AP}| = \frac{\sqrt{5}}{2}$, then the area of the triangle ABP is:

- (A) $\frac{3}{2}$ (B) $\frac{\sqrt{30}}{4}$ (C) $\frac{\sqrt{26}}{4}$ (D) 2

Q73. JEE Main 2026

Let PQR be a triangle such that $\vec{PQ} = -2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{PR} = a\hat{i} + b\hat{j} - 4\hat{k}$, $a, b \in \mathbb{Z}$. Let S be the point on QR, which is equidistant from the lines PQ and PR. If $|\vec{PS}| = 9$ and $\vec{PS} = \hat{i} - 7\hat{j} + 2\hat{k}$, then the value of $3a - 4b$ is ____

Q74. JEE Main 2025

Let A, B, C be three points in xy-plane, whose position vector are given by $\sqrt{3}\hat{i} + \hat{j}$, $\hat{i} + \sqrt{3}\hat{j}$ and $a\hat{i} + (1-a)\hat{j}$ respectively with respect to the origin O. If the distance of the point C from the line bisecting the angle between the vectors $\vec{OA}$ and $\vec{OB}$ is $\frac{9}{\sqrt{2}}$, then the sum of all the possible values of a is:

- (A) 2
(B) 0
(C) $\frac{9}{2}$
(D) 1

Q75. JEE Main 2024

The position vectors of the vertices A, B and C of a triangle are $2\hat{i} - 3\hat{j} + 3\hat{k}$, $2\hat{i} + 2\hat{j} + 3\hat{k}$ and $-\hat{i} + \hat{j} + 3\hat{k}$ respectively. Let l denotes the length of the angle bisector AD of $\angle BAC$ where D is on the line segment BC, then $2l^2$ equals:

- (A) 45 (B) 50 (C) 42 (D) 49

Q76. JEE Main 2020

A vector $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$ ($\alpha, \beta \in \mathbb{R}$) lies in the plane of the vectors, $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \hat{i} - \hat{j} + 4\hat{k}$. If $\vec{a}$ bisects the angle between $\vec{b}$ and $\vec{c}$, then:

- (A) $\vec{a} \cdot \hat{i} + 3 = 0$ (B) $\vec{a} \cdot \hat{k} - 4 = 0$
(C) $\vec{a} \cdot \hat{i} + 1 = 0$ (D) $\vec{a} \cdot \hat{k} + 2 = 0$

Q77. JEE Main 2019

Let $\sqrt{3}\hat{i} + \hat{j}$, $\hat{i} + \sqrt{3}\hat{j}$ and $\beta\hat{i} + (1-\beta)\hat{j}$ respectively be the position vectors of the points A, B and C with respect to the origin O. If the distance of C from the bisector of the acute angle between OA and OB is $\frac{3}{\sqrt{2}}$, then the sum of all possible values of β is:

- (A) 4 (B) 1 (C) 2 (D) 3

Q78. JEE Main 2018

If the position vectors of the vertices A, B and C of a ΔABC are respectively $4\hat{i} + 7\hat{j} + 8\hat{k}$, $2\hat{i} + 3\hat{j} + 4\hat{k}$, and $2\hat{i} + 5\hat{j} + 7\hat{k}$, then the position vectors of the point, where the bisector of $\angle A$ meets BC is:

- (A) $\frac{1}{2}(4\hat{i} + 8\hat{j} + 11\hat{k})$
(B) $\frac{1}{3}(6\hat{i} + 11\hat{j} + 15\hat{k})$
(C) $\frac{1}{3}(6\hat{i} + 13\hat{j} + 18\hat{k})$
(D) $\frac{1}{4}(8\hat{i} + 14\hat{j} + 19\hat{k})$

Q79. JEE Main 2008

The vector $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$ lies in the plane of the vectors $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \hat{j} + \hat{k}$ and bisects the angle between $\vec{b}$ and $\vec{c}$. Then which one of the following gives possible values of α and β ?

- (A) $\alpha = 2, \beta = 2$ (B) $\alpha = 1, \beta = 2$
(C) $\alpha = 2, \beta = 1$ (D) $\alpha = 1, \beta = 1$

Topic 9: Position Vector

Q80. JEE Main 2026

Let O be the origin, $\overrightarrow{OP} = \vec{a}$ and $\overrightarrow{OQ} = \vec{b}$. If R is the point on $\overrightarrow{OP}$ such that $\overrightarrow{OR} = 5\overrightarrow{OR}$, and M is the point such that $\overrightarrow{OQ} = 5\overrightarrow{RM}$, then $\overrightarrow{PM}$ is equal to :

- (A) $\frac{1}{5}(\vec{a} - 4\vec{b})$ (B) $\frac{1}{5}(\vec{b} - 4\vec{a})$
(C) $\frac{1}{5}(-\vec{a} + 4\vec{b})$ (D) $\frac{1}{5}(-\vec{b} + 4\vec{a})$

Q81. JEE Main 2023

Let $ABCD$ be a quadrilateral. If E and F are the mid points of the diagonals AC and BD respectively and $(\overrightarrow{AB} - \overrightarrow{BC}) + (\overrightarrow{AD} - \overrightarrow{DC}) = k\overrightarrow{FE}$, then k is equal to :

- (A) -2 (B) 4
(C) -4 (D) 2

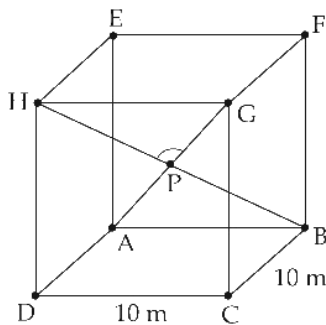
Q82. JEE Main 2023

An arc PQ of a circle subtends a right angle at its centre O . The mid point of the arc PQ is R . If $\overrightarrow{OP} = \vec{u}$, $\overrightarrow{OR} = \vec{v}$, and $\overrightarrow{OQ} = \alpha\vec{u} + \beta\vec{v}$, then α, β are the roots of the equation :

- (A) $x^2 + x - 2 = 0$
(B) $3x^2 + 2x - 1 = 0$
(C) $3x^2 - 2x - 1 = 0$
(D) $x^2 - x - 2 = 0$

Q83. JEE Main 2021

A hall has a square floor of dimension $10 \text{ m} \times 10 \text{ m}$ (see the figure) and vertical walls. If the angle GPH between the diagonals AG and BH is $\cos^{-1}\frac{1}{5}$, then the height of the hall (in meters) is :



- (A) 5 (B) $2\sqrt{10}$ (C) $5\sqrt{3}$ (D) $5\sqrt{2}$

Q84. JEE Main 2020

Let the position vectors of points 'A' and 'B' be $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} + \hat{j} + 3\hat{k}$, respectively. A point 'P' divides the line segment AB internally in the ratio

$\lambda : 1$ ($\lambda > 0$). If O is the origin and $\overrightarrow{OB} \cdot \overrightarrow{OP} - 3|\overrightarrow{OA} \times \overrightarrow{OP}|^2 = 6$, then

λ is equal to _____.

Q85. JEE Main 2016

In a triangle ABC , right angled at the vertex A , if the position vectors of A, B and C are respectively $3\hat{i} + \hat{j} - \hat{k}$, $-\hat{i} + 3\hat{j} + p\hat{k}$ and $5\hat{i} + q\hat{j} - 4\hat{k}$, then the point (p, q) lies on a line :

- (A) parallel to x-axis.
(B) parallel to y-axis.
(C) making an acute angle with the positive direction of x-axis.
(D) making an obtuse angle with the positive direction of x-axis.

Q86. JEE Main 2013

If the vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC , then the length of the median through A is :

- (A) $\sqrt{18}$ (B) $\sqrt{72}$ (C) $\sqrt{33}$ (D) $\sqrt{45}$

Q87. JEE Main 2006

The values of a , for which the points A, B, C with position vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $a\hat{i} - 3\hat{j} + \hat{k}$ respectively are the vertices of a right angled triangle with $C = \frac{\pi}{2}$ are :

- (A) 2 and 1 (B) -2 and -1
(C) -2 and 1 (D) 2 and -1

Q88. JEE Main 2003

The vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ & $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of triangle ABC . The length of the median through A is :

- (A) $\sqrt{288}$ (B) $\sqrt{18}$ (C) $\sqrt{72}$ (D) $\sqrt{33}$

Q89. JEE Main 2003

A tetrahedron has vertices at $O(0, 0, 0)$, $A(1, 2, 1)$, $B(2, 1, 3)$ and $C(-1, 1, 2)$. Then the angle between the faces OAB and ABC will be :

- (A) 90°
(B) $\cos^{-1}\left(\frac{19}{35}\right)$
(C) $\cos^{-1}\left(\frac{17}{31}\right)$
(D) 30°

Topic 10: Centre of Triangle

Q90. JEE Main 2025

Let the position vectors of three vertices of a triangle be $4\vec{p} + \vec{q} - 3\vec{r}$, $-5\vec{p} + \vec{q} + 2\vec{r}$ and $2\vec{p} - \vec{q} + 2\vec{r}$. If the position vectors of the orthocenter and the circumcenter of the triangle are $\frac{\vec{p} + \vec{q} + \vec{r}}{4}$ and $\alpha\vec{p} + \beta\vec{q} + \gamma\vec{r}$ respectively, then $\alpha + 2\beta + 5\gamma$ is equal to :

- (A) 4 (B) 3 (C) 1 (D) 6

Q91. JEE Main 2025

Let the three sides of a triangle ABC be given by the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$. Let G be the centroid of the triangle ABC . Then $6(|\overrightarrow{AG}|^2 + |\overrightarrow{BG}|^2 + |\overrightarrow{CG}|^2)$ is equal to _____.

Q92. JEE Main 2024

Let the position vectors of the vertices A, B and C of a triangle be $2\hat{i} + 2\hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} + 2\hat{k}$ and $2\hat{i} + \hat{j} + 2\hat{k}$ respectively. Let l_1, l_2 and l_3 be the lengths of perpendiculars drawn from the ortho center of the triangle on the sides AB, BC and CA respectively, then $l_1^2 + l_2^2 + l_3^2$ equals:

- (A) $\frac{1}{4}$ (B) $\frac{1}{5}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$

Q93. JEE Main 2019

Let A (3, 0, -1), B(2, 10, 6) and C(1, 2, 1) be the vertices of a triangle and M be the midpoint of AC. If G divides BM in the ratio, 2 : 1, then $\cos(\angle GOA)$ (O being the origin) is equal to :

- (A) $\frac{1}{\sqrt{15}}$ (B) $\frac{1}{6\sqrt{10}}$
(C) $\frac{1}{\sqrt{30}}$ (D) $\frac{1}{2\sqrt{15}}$

Q94. JEE Main 2016

Let ABC be a triangle whose circumcentre is at P. If the position vectors of A, B, C and P are $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\frac{\vec{a} + \vec{b} + \vec{c}}{4}$ respectively, then the position vector of the orthocentre of this triangle, is :

- (A) $\vec{a} + \vec{b} + \vec{c}$
(B) $-\left(\frac{\vec{a} + \vec{b} + \vec{c}}{2}\right)$
(C) $\vec{0}$
(D) $\left(\frac{\vec{a} + \vec{b} + \vec{c}}{2}\right)$

Q95. JEE Main 2023

If the points P and Q are respectively the circumcenter and the orthocentre of a $\triangle ABC$, then $\vec{PA} + \vec{PB} + \vec{PC}$ is equal to :

- (A) $\vec{QP}$
(B) $\vec{PQ}$
(C) $2\vec{PQ}$
(D) $2\vec{QP}$

Topic 11: Vector Equation Take Dot/Cross**Q96. JEE Main 2023**

Let $\vec{u} = \hat{i} - \hat{j} - 2\hat{k}$, $\vec{v} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{v} \cdot \vec{w} = 2$ and $\vec{v} \times \vec{w} = \vec{u} + \lambda\vec{v}$. Then $\vec{u} \cdot \vec{w}$ is equal to :

- (A) $-\frac{2}{3}$ (B) $\frac{3}{2}$ (C) 2 (D) 1

Q97. JEE Main 2024

Let $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}$ and a vector $\vec{c}$ be such that $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times \vec{c} = \hat{i} + 8\hat{j} + 13\hat{k}$. If $\vec{a} \cdot \vec{c} = 13$, then $(24 - \vec{b} \cdot \vec{c})$ is equal to _____.

Q98. JEE Main 2023

Let $\vec{a}$ and $\vec{b}$ be two vectors, Let $|\vec{a}| = 1$, $|\vec{b}| = 4$ and $\vec{a} \cdot \vec{b} = 2$. If $\vec{c} = (2\vec{a} \times \vec{b}) - 3\vec{b}$, then the value of $\vec{b} \cdot \vec{c}$ is :

- (A) -48 (B) -60 (C) -84 (D) -24

Q99. JEE Main 2022

Let ABC be a triangle such that $\vec{BC} = \vec{a}$, $\vec{CA} = \vec{b}$, $\vec{AB} = \vec{c}$, $|\vec{a}| = 6\sqrt{2}$, $|\vec{b}| = 2\sqrt{3}$ and $\vec{b} \cdot \vec{c} = 12$. Consider the statements :

- (S1) : $|(\vec{a} \times \vec{b}) + (\vec{c} \times \vec{b})| - |\vec{c}| = 6(2\sqrt{2} - 1)$
(S2) : $\angle ACB = \cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$

Then

- (A) both (S1) and (S2) are true
(B) only (S1) is true
(C) only (S2) is true
(D) both (S1) and (S2) are false

Q100. JEE Main 2022

Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three non-coplanar vectors such that $\vec{a} \times \vec{b} = 4\vec{c}$, $\vec{b} \times \vec{c} = 9\vec{a}$ and $\vec{c} \times \vec{a} = \alpha\vec{b}$, $\alpha > 0$. If $|\vec{a}| + |\vec{b}| + |\vec{c}| = \frac{1}{36}$, then α is equal to _____.

Q101. JEE Main 2022

Let $\vec{a}$ and $\vec{b}$ be the vectors along the diagonals of a parallelogram having area $2\sqrt{2}$. Let the angle between $\vec{a}$ and $\vec{b}$ be acute, $|\vec{a}| = 1$, and $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$. If $\vec{c} = 2\sqrt{2}(\vec{a} \times \vec{b}) - 2\vec{b}$, then an angle between $\vec{b}$ and $\vec{c}$ is :

- (A) $\frac{\pi}{4}$ (B) $-\frac{\pi}{4}$
(C) $\frac{5\pi}{6}$ (D) $\frac{3\pi}{4}$

Q102. JEE Main 2011

The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying $\vec{b} \times \vec{c} = \vec{b} \times \vec{d}$ and $\vec{a} \cdot \vec{d} = 0$. Then the vector $\vec{d}$ is equal to :

- (A) $\vec{c} + \left(\frac{\vec{a} \cdot \vec{c}}{\vec{a} \cdot \vec{b}}\right)\vec{b}$
(B) $\vec{b} + \left(\frac{\vec{b} \cdot \vec{c}}{\vec{a} \cdot \vec{b}}\right)\vec{c}$
(C) $\vec{c} - \left(\frac{\vec{a} \cdot \vec{c}}{\vec{a} \cdot \vec{b}}\right)\vec{b}$
(D) $\vec{b} - \left(\frac{\vec{b} \cdot \vec{c}}{\vec{a} \cdot \vec{b}}\right)\vec{c}$

Q103. JEE Main 2025

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{d} = \vec{a} \times \vec{b}$. If $\vec{c}$ is a vector such that $\vec{a} \cdot \vec{c} = |\vec{c}|$, $|\vec{c} - 2\vec{d}|^2 = 8$ and the angle between $\vec{d}$ and $\vec{c}$ is $\frac{\pi}{4}$, then $|10 - 3\vec{b} \cdot \vec{c}| + |\vec{d} \times \vec{c}|^2$ is equal to _____.

Q104. JEE Main 2026

Let $\hat{u}$ and $\hat{v}$ be unit vectors inclined at an acute angle such that $|\hat{u} \times \hat{v}| = \frac{\sqrt{3}}{2}$. If $\vec{A} = \lambda\hat{u} + \hat{v} + (\hat{u} \times \hat{v})$, then λ is equal to:

- (A) $\frac{4}{3}(\vec{A} \cdot \hat{u}) - \frac{2}{3}(\vec{A} \cdot \hat{v})$
(B) $\frac{2}{3}(\vec{A} \cdot \hat{u}) - \frac{1}{3}(\vec{A} \cdot \hat{v})$
(C) $\frac{4}{3}(\vec{A} \cdot \hat{u}) + \frac{2}{3}(\vec{A} \cdot \hat{v})$
(D) $(\vec{A} \cdot \hat{u}) - \frac{1}{2}(\vec{A} \cdot \hat{v})$

Topic 12: Rotation of Vector

Q105. JEE Main 2026

Two adjacent sides of a parallelogram PQRS are given by $\overrightarrow{PQ} = \hat{j} + \hat{k}$ and $\overrightarrow{PS} = \hat{i} - \hat{j}$. If the side PS is rotated about the point P by an acute angle α in the plane of the parallelogram so that it becomes perpendicular to the side PQ, then $\sin^2\left(\frac{5\alpha}{2}\right) - \sin^2\left(\frac{\alpha}{2}\right)$ is equal to:

- (A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{\sqrt{3}}{4}$ (D) $\frac{2\sqrt{3}}{5}$

Q106. JEE Main 2023

The vector $\vec{a} = -\hat{i} + 2\hat{j} + \hat{k}$ is rotated through a right angle, passing through the y-axis in its way and the resulting vector is $\vec{b}$. Then the projection of $3\vec{a} + \sqrt{2}\vec{b}$ on $\vec{c} = 5\hat{i} + 4\hat{j} + 3\hat{k}$ is:

- (A) $\sqrt{6}$ (B) $2\sqrt{3}$ (C) 1 (D) $3\sqrt{2}$

Q107. JEE Main 2021

For $p > 0$, a vector $\vec{v}_2 = 2\hat{i} + (p+1)\hat{j}$ is obtained by rotating the vector $\vec{v}_1 = \sqrt{3}p\hat{i} + \hat{j}$ by an angle θ about origin in counter clockwise direction. If $\tan \theta = \frac{(\alpha\sqrt{3}-2)}{(4\sqrt{3}+3)}$, then the value of α is equal to _____.

Q108. JEE Main 2021

A vector $\vec{a}$ has components $3p$ and 1 with respect to a rectangular cartesian system. This system is rotated through a certain angle about the origin in the counter clockwise sense. If, with respect to new system, $\vec{a}$ has components $p+1$ and $\sqrt{10}$, then the value of p is equal to:

- (A) 1 (B) $-\frac{5}{4}$ (C) $\frac{4}{5}$ (D) -1

Q109. JEE Main 2021

Let a vector $\alpha\hat{i} + \beta\hat{j}$ be obtained by rotating the vector $\sqrt{3}\hat{i} + \hat{j}$ by an angle 45° about the origin in counterclockwise direction in the first quadrant. Then the area of triangle having vertices (α, β) , $(0, \beta)$ and $(0, 0)$ is equal to:

- (A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$ (C) 1 (D) $2\sqrt{2}$

Topic 13: Miscellaneous

Q110. JEE Main 2026

Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \lambda\hat{j} + 2\hat{k}$, $\lambda \in \mathbb{Z}$ be two vectors. Let $\vec{c} = \vec{a} \times \vec{b}$ and $\vec{d}$ be a vector of magnitude 2 in yz -plane. If $|\vec{c}| = \sqrt{53}$, then the maximum possible value of $(\vec{c} \cdot \vec{d})^2$ is equal to:

- (A) 52 (B) 26 (C) 104 (D) 208

Q111. JEE Main 2025

Let the arc AC of a circle subtend a right angle at the centre O . If the point B on the arc AC , divides the arc AC such that $\frac{\text{length of arc } AB}{\text{length of arc } BC} = \frac{1}{5}$, and $\overrightarrow{OC} = \alpha\overrightarrow{OA} + \beta\overrightarrow{OB}$, then $\alpha + \sqrt{2}(\sqrt{3}-1)\beta$ is equal to

- (A) $2\sqrt{3}$ (B) $5\sqrt{3}$ (C) $2 + \sqrt{3}$ (D) $2 - \sqrt{3}$

Q112. JEE Main 2023

For any vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, with $10|a_i| < 1$, $i = 1, 2, 3$, consider the following statements:

- (A): $\max\{|a_1|, |a_2|, |a_3|\} \leq |\vec{a}|$
(B): $|\vec{a}| \leq 3 \max\{|a_1|, |a_2|, |a_3|\}$

- (A) Only (B) is true
(B) Only (A) is true
(C) Neither (A) nor (B) is true
(D) Both (A) and (B) are true

Q113. JEE Main 2022

Let a vector $\vec{a}$ has magnitude 9. Let a vector $\vec{b}$ be such that for every $(x, y) \in \mathbb{R} \times \mathbb{R} - \{(0, 0)\}$, the vector $(x\vec{a} + y\vec{b})$ is perpendicular to the vector $(6y\vec{a} - 18x\vec{b})$. Then the value of $|\vec{a} \times \vec{b}|$ is equal to:

- (A) $9\sqrt{3}$
(B) $27\sqrt{3}$
(C) 9
(D) 81

Q114. JEE Main 2022

Let $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} - 3\hat{j} + 2\hat{k}$. Then the number of vectors $\vec{b}$ such that $\vec{b} \times \vec{c} = \vec{a}$ and $|\vec{b}| \in \{1, 2, \dots, 10\}$ is:

- (A) 0 (B) 1 (C) 2 (D) 3

Q115. JEE Main 2021

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b}$ and $\vec{c} = \hat{j} - \hat{k}$ be three vectors such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \cdot \vec{b} = 1$. If the length of projection vector of the vector $\vec{b}$ on the vector $\vec{a} \times \vec{c}$ is 1, then the value of $3l^2$ is equal to _____.

Q116. JEE Main 2021

Let $\vec{a}$, $\vec{b}$, $\vec{c}$ three vectors mutually perpendicular to each other and have same magnitude. If a vector $\vec{r}$ satisfies $\vec{a} \times \{(\vec{r} - \vec{b}) \times \vec{a}\} + \vec{b} \times \{(\vec{r} - \vec{c}) \times \vec{b}\} + \vec{c} \times \{(\vec{r} - \vec{a}) \times \vec{c}\} = \vec{0}$, then $\vec{r}$ is equal to:

- (A) $\frac{1}{3}(\vec{a} + \vec{b} + \vec{c})$
(B) $\frac{1}{3}(2\vec{a} + \vec{b} - \vec{c})$
(C) $\frac{1}{2}(\vec{a} + \vec{b} + \vec{c})$
(D) $\frac{1}{2}(\vec{a} + \vec{b} + 2\vec{c})$

Q117. JEE Main 2020

Let $a, b, c \in \mathbb{R}$ be such that $a^2 + b^2 + c^2 = 1$. If $a \cos \theta = b \cos\left(\theta + \frac{2\pi}{3}\right) = c \cos\left(\theta + \frac{4\pi}{3}\right)$, where $\theta = \frac{\pi}{9}$, then the angle between the vectors $a\hat{i} + b\hat{j} + c\hat{k}$ and $b\hat{i} + c\hat{j} + a\hat{k}$ is:

- (A) 0 (B) $\frac{\pi}{9}$ (C) $\frac{2\pi}{3}$ (D) $\frac{\pi}{2}$

