A Premier Institute for Pre-Medical & Pre Engineering
SRI VIDYA ARADHANA ACADEMY
"Transforming Your DREAMS Into Reality...!"
PCMB — PHYSICS CHEMISTRY MATHEMATICS BIOLOGY
NEET/JEE

Topic: Basic Mathematics

Mathematics **Assignment 07 • Intervals & Linear Inequalities** Chetan Sir

"Confidence in mathematics is not given — it is built, one solved problem at a time."

An **interval** is a set of all real numbers lying between two given values — with **no gaps**. [ ] means the end point is **included**; ( ) means it is **excluded**. ∞ is not a real number, so we **always** take an open bracket at ∞.

Rule 1 (add/subtract k): sign stays. **Rule 2** (multiply/divide by k): stays if $k > 0$, **reverses if** $k < 0$. **Rule 3**: never multiply by an expression of **unknown sign**. **Rule 4** (reciprocal, same sign): sign **reverses**.

Interval algebra — if $x \in [a, b]$:

Operation	0 **outside** $[a, b]$	0 **inside** $[a, b]$
x^2	$[\min(a^2, b^2), \max(a^2, b^2)]$	$[0, \max(a^2, b^2)]$
$\frac{1}{x}$	$\left[\frac{1}{b}, \frac{1}{a}\right]$ (order flips)	$\left(-\infty, \frac{1}{a}\right] \cup \left[\frac{1}{b}, \infty\right)$ (splits)
$\sqrt{x}$	keep $x \geq 0$, then $[\sqrt{a}, \sqrt{b}]$; if $b < 0$ then $\varnothing$	

SECTION A : TYPES OF INTERVALS

1. Classify each as open, closed, or open–closed/closed–open, and state whether finite or infinite:
(i) $(-7, 2]$ (ii) $[3, 3]$ (iii) $(-\infty, 6)$ (iv) $[5, \infty)$

2. True or false: $(4, 4) = \varnothing$.

3. Write in interval notation: all real numbers except 0.

SECTION B : UNION & INTERSECTION

4. Find $[2, 9] \cup [5, 12]$.

5. Find $(-6, 3) \cap [-1, 8]$.

6. Find $(2, 5) \cap (7, 10)$.

7. Find $(-\infty, 4) \cup [4, \infty)$.

8. Find $[-5, 3] \cap [3, 9)$.

9. Find $[-8, 6] \cap (0, 10) \cap [-3, 4]$.

10. Find $[-6, 2] \cup (0, 9) \cup [7, 12)$.

SECTION C : INTERVAL ALGEBRA

11. Find the range of x^2 for:
(i) $x \geq 2$ (ii) $x < -4$ (iii) $x > 5$

12. Find the range of x^2 for:
(i) $x \leq -3$ (ii) $[-5, 2]$ (iii) $[-8, -3]$

13. Find the range of x^2 for:
(i) $(-3, 4]$ (ii) $[-6, -1]$

14. Find the range of $\frac{1}{x}$ for:
(i) $x > 3$ (ii) $x < -2$ (iii) $x \geq 4$

15. Find the range of $\frac{1}{x}$ for:
(i) $x \leq -5$ (ii) $x \in (-4, 5) - \{0\}$ (iii) $[2, 6]$

16. Find the range of $\frac{1}{x}$ for:
(i) $(-\infty, -3]$ (ii) $[-10, -2]$

17. Find the range of $\sqrt{x}$ for:
(i) $[9, 49]$ (ii) $[-16, 25]$ (iii) $[-25, -9]$

SECTION D : RULES OF INEQUALITIES

18. Given $6 > 2$, what does each operation give?
(i) add 5 to both sides (ii) multiply both sides by 3
(iii) multiply both sides by -3 (iv) divide both sides by -2

19. True or false: $a > b \Rightarrow a^2 > b^2$. Justify with an example.

SECTION E : SOLVING LINEAR INEQUALITIES

20. Solve $4x - 5 > 3x + 7$.

21. Solve $3 - 2x \leq 11$.

22. Solve $\frac{2x - 1}{3} \geq \frac{x + 4}{2}$.

23. Solve $5(x-2) < 3(x+4)$.

SECTION F : COMPOUND & SYSTEMS

24. Solve $-4 < 2x - 6 \leq 8$.

25. Solve $3x - 2 > 1$ **and** $5 - x \geq 0$.

26. Solve $2x + 3 < 5$ **or** $x - 1 > 4$.

27. Find the number of integral solutions of $-6 < 3x - 3 \leq 9$.

28. Find the number of integral solutions of $-8 \leq 5 - 3x < 10$.

29. Find the number of integral solutions of $-9 < \dfrac{4-3x}{2} \leq 6$.

30. Find the greatest integer x satisfying $\dfrac{3x+1}{2} \leq \dfrac{2x-1}{3} + 5$.

SECTION G : WORD PROBLEMS

31. Find all pairs of consecutive odd natural numbers, both greater than 8, such that their sum is less than 36.

32. Ravi scored 68 and 74 marks in his first two unit tests. Find the minimum marks he must score in the third test to have an average of at least 70.

33. The longest side of a triangle is twice its shortest side, and the third side is 3 cm longer than the shortest side. If the perimeter is at most 53 cm, find the maximum possible length of the shortest side.

SECTION H : MULTIPLE CHOICE

34. The solution of $2x - 3 < 7$ is

(A) $x < 5$ (B) $x > 5$ (C) $x \leq 5$ (D) $x \geq 5$

35. If $x \in [-3, 6]$, then x^2 lies in

(A) $[9, 36]$ (B) $[0, 36]$ (C) $[-9, 36]$ (D) $[0, 9]$

ANSWER KEY

1. (i) open–closed, finite (ii) closed, finite $\{3\}$ (iii) open, infinite (iv) closed, infinite
2. True
3. $(-\infty, 0) \cup (0, \infty)$, i.e. $\mathbb{R} - \{0\}$ or $\mathbb{R}_0$
4. $[2, 12]$
5. $[-1, 3)$
6. $\varnothing$
7. $(-\infty, \infty) = \mathbb{R}$
8. $\{3\}$
9. $(0, 4]$
10. $[-6, 12)$
11. (i)$[4, \infty)$ (ii)$(16, \infty)$ (iii)$(25, \infty)$
12. (i)$[9, \infty)$ (ii)$[0, 25]$ (iii)$[9, 64]$
13. (i)$[0, 16]$ (ii)$[1, 36]$
14. (i)$(0, \frac{1}{3})$ (ii)$(-\frac{1}{2}, 0)$ (iii)$(0, \frac{1}{4}]$
15. (i)$[-\frac{1}{5}, 0)$ (ii)$(-\infty, -\frac{1}{4}) \cup (\frac{1}{5}, \infty)$ (iii)$[\frac{1}{6}, \frac{1}{2}]$
16. (i)$[-\frac{1}{3}, 0)$ (ii)$[-\frac{1}{2}, -\frac{1}{10}]$
17. (i)$[3, 7]$ (ii)$[0, 5]$ (iii)$\varnothing$
18. (i)$11 > 7$ (ii)$18 > 6$ (iii)$-18 < -6$ (iv)$-3 < -1$
19. False; e.g. $a = 1, b = -5$: $a > b$ but $a^2 < b^2$
20. $x > 12$
21. $x \geq -4$
22. $x \geq 14$
23. $x < 11$
24. $1 < x \leq 7$
25. $1 < x \leq 5$
26. $(-\infty, 1) \cup (5, \infty)$
27. 5 integers $\{0, 1, 2, 3, 4\}$
28. 6 integers $\{-1, 0, 1, 2, 3, 4\}$
29. 10 integers $\{-2, \ldots, 7\}$
30. greatest integer $= 5$
31. $(9, 11), (11, 13), (13, 15), (15, 17)$
32. 68 marks
33. 12.5 cm
34. A
35. B