

HI EVERYONE,

THE REAL LEARNING IN MATHEMATICS HAPPENS WHEN YOU ACTIVELY ENGAGE WITH A PROBLEM, EXPLORE DIFFERENT METHODS, AND WORK THROUGH CHALLENGES. THEREFORE, WE STRONGLY ENCOURAGE YOU TO USE THIS SOLUTION KEY RESPONSIBLY.

PLEASE ATTEMPT ALL THE PROBLEMS ON YOUR OWN FIRST, GIVING THEM YOUR BEST AND MOST HONEST EFFORT. THESE SOLUTIONS ARE TO HELP YOU GET UNSTUCK ON A PROBLEM AFTER YOU HAVE ALREADY TRIED YOUR BEST.

YOUR EFFORT AND DEDICATION ARE THE TRUE KEYS TO SUCCESS.

Sets • Assignment 01 • Solutions

Topic: Representation & Types of Sets

Sub: Mathematics

11th • JEE

Chetan Sir

1. **[Well-defined sets]** Which of the following are well-defined sets? Justify. (i) the vowels in the English alphabet (ii) a collection of books (iii) the rivers of India (iv) the ten most talented writers of India (v) the collection of all even integers (vi) the difficult chapters in Mathematics. [NCERT]

Solution: A collection is a *set* only when membership is decided by a clear, objective rule with no room for opinion.

- (i) Well-defined — the vowels are exactly a, e, i, o, u . ✓
- (ii) *Not* a set — “books” is not specified; which books? ✗
- (iii) Well-defined — the rivers of India form a definite list. ✓
- (iv) *Not* a set — “most talented” is a matter of opinion. ✗
- (v) Well-defined — divisibility by 2 is an objective test. ✓
- (vi) *Not* a set — “difficult” varies from person to person. ✗

Answer: The well-defined sets are (i), (iii) and (v).

2. **[Well-defined sets]** Which of the following are sets? (i) novels written by Munshi Premchand (ii) the most dangerous animals of the world (iii) all the questions in this chapter (iv) all boys in your class. [NCERT]

Solution: Apply the same test — is membership objective?

- (i) A set — Premchand’s novels form a fixed list. ✓
- (ii) *Not* a set — “most dangerous” is subjective. ✗
- (iii) A set — the questions in the chapter are definite. ✓
- (iv) A set — the boys in your class are clearly identified. ✓

Answer: The sets are (i), (iii) and (iv).

3. **[Roster form]** Write the solution set of $x^2 + x - 2 = 0$ in roster form. [NCERT]

Solution: Factorise and solve, then list the roots.

$$\begin{aligned}x^2 + x - 2 &= 0 \\(x + 2)(x - 1) &= 0 \\x &= -2 \quad \text{or} \quad x = 1.\end{aligned}$$

Answer: Solution set = $\{-2, 1\}$.

4. [Roster form] Write $\{x : x \text{ is a positive integer and } x^2 < 40\}$ in roster form.

[NCERT]

Solution: We need positive integers whose square is below 40.

$$1^2 = 1, 2^2 = 4, 3^2 = 9, 4^2 = 16, 5^2 = 25, 6^2 = 36 (< 40), \\ 7^2 = 49 (> 40).$$

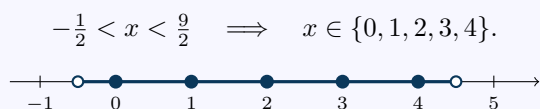
So x can be 1, 2, 3, 4, 5, 6.

Answer: $\{1, 2, 3, 4, 5, 6\}$.

5. [Roster form] Write $\{x : x \text{ is an integer and } -\frac{1}{2} < x < \frac{9}{2}\}$ in roster form.

[NCERT]

Solution: We list the integers strictly between -0.5 and 4.5 :



Answer: $\{0, 1, 2, 3, 4\}$.

6. [Roster form] Write the set of all two-digit natural numbers whose digits add up to 8.

[NCERT]

Solution: Let the number be $\overline{ab}$ with $a + b = 8$, where $a \in \{1, \dots, 9\}$ (tens digit, non-zero) and $b \in \{0, \dots, 9\}$.

$$a = 1, b = 7 \rightarrow 17$$

$$a = 2, b = 6 \rightarrow 26$$

$$a = 3, b = 5 \rightarrow 35$$

$$a = 4, b = 4 \rightarrow 44$$

$$a = 5, b = 3 \rightarrow 53$$

$$a = 6, b = 2 \rightarrow 62$$

$$a = 7, b = 1 \rightarrow 71$$

$$a = 8, b = 0 \rightarrow 80.$$

Answer: $\{17, 26, 35, 44, 53, 62, 71, 80\}$.

7. [Roster form] Write $\{x : x^2 = x, x \in \mathbb{N}\}$ in roster form.

[NCERT]

Solution:

$$x^2 = x \\ x^2 - x = 0 \\ x(x - 1) = 0 \implies x = 0 \text{ or } x = 1.$$

Since $0 \notin \mathbb{N}$, only $x = 1$ is admissible.

Answer: $\{1\}$.

8. [Roster form] Write the set of all distinct letters of the word MATHEMATICS.

Solution: List the letters, keeping each one only once (in a set, repetition is ignored):

$$\text{MATHEMATICS} \longrightarrow \{M, A, T, H, E, I, C, S\}.$$

Answer: $\{M, A, T, H, E, I, C, S\}$ (8 distinct letters).

9. [Roster form] Write $\left\{x : \frac{x-1}{x+3} = 4, x \in \mathbb{R}\right\}$ in roster form.

[NCERT]

Solution: Solve the equation (with $x \neq -3$):

$$\begin{aligned}\frac{x-1}{x+3} &= 4 \\ x-1 &= 4(x+3) \\ x-1 &= 4x+12 \\ -13 &= 3x \\ x &= -\frac{13}{3}.\end{aligned}$$

This is allowed since $-\frac{13}{3} \neq -3$.

Answer: $\left\{-\frac{13}{3}\right\}$.

10. **[Set-builder form]** Express $\{1, 4, 9, 16, 25, \dots\}$ in set-builder form.

[NCERT]

Solution: Each term is a perfect square: $1 = 1^2$, $4 = 2^2$, $9 = 3^2, \dots$

Answer: $\{x : x = n^2, n \in \mathbb{N}\}$.

11. **[Set-builder form]** Express $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}\right\}$ in set-builder form.

[NCERT]

Solution: The n -th term has numerator n and denominator $n+1$, for $n = 1, 2, \dots, 6$.

Answer: $\left\{x : x = \frac{n}{n+1}, n \in \mathbb{N}, 1 \leq n \leq 6\right\}$.

12. **[Set-builder form]** Express $\{2, 4, 8, 16, 32\}$ in set-builder form.

[NCERT]

Solution: $2 = 2^1$, $4 = 2^2$, $8 = 2^3$, $16 = 2^4$, $32 = 2^5$ — powers of 2.

Answer: $\{x : x = 2^n, n \in \mathbb{N}, 1 \leq n \leq 5\}$.

13. **[Set-builder form]** Express $\{3, 6, 9, 12\}$ in set-builder form.

[NCERT]

Solution: Each term is a multiple of 3: $3 \cdot 1$, $3 \cdot 2$, $3 \cdot 3$, $3 \cdot 4$.

Answer: $\{x : x = 3n, n \in \mathbb{N}, 1 \leq n \leq 4\}$.

14. **[Set-builder form]** Express $\{5, 25, 125, 625\}$ in set-builder form.

Solution: $5 = 5^1$, $25 = 5^2$, $125 = 5^3$, $625 = 5^4$ — powers of 5.

Answer: $\{x : x = 5^n, n \in \mathbb{N}, 1 \leq n \leq 4\}$.

15. **[Empty / finite / infinite]** Which of $A = \{x \in \mathbb{R} : x^2 + 1 = 0\}$, $B = \{x \in \mathbb{R} : x^2 - 16 = 0\}$, $C = \{x \in \mathbb{R} : x^2 = x + 2\}$ are empty sets?

[NCERT]

Solution: Solve each over $\mathbb{R}$.

$$\begin{aligned}A : \quad x^2 + 1 = 0 &\Rightarrow x^2 = -1 \\ B : \quad x^2 - 16 = 0 &\Rightarrow x = \pm 4\end{aligned}$$

$$\begin{aligned}\text{no real root} &\Rightarrow A = \emptyset. \\ B &= \{-4, 4\} \neq \emptyset.\end{aligned}$$

$$C : x^2 - x - 2 = 0 \Rightarrow (x - 2)(x + 1) = 0$$

$$C = \{2, -1\} \neq \emptyset.$$

Answer: Only A is the empty set.

16. **[Classify]** Classify $\{x : x^4 - x^3 - x^2 = 0, x \in \mathbb{N}\}$ as null, singleton, finite or infinite.

[NCERT]

Solution: Factorise the polynomial.

$$\begin{aligned} x^4 - x^3 - x^2 &= 0 \\ x^2(x^2 - x - 1) &= 0 \\ x = 0 \quad \text{or} \quad x &= \frac{1 \pm \sqrt{5}}{2}. \end{aligned}$$

The roots are 0 and $\frac{1 \pm \sqrt{5}}{2} \approx 1.618, -0.618$ — *none* is a natural number ($0 \notin \mathbb{N}$).

Answer: The set is the **null set** $\emptyset$.

17. **[Finite / infinite]** State which are finite and which are infinite: (i) the set of days of the week; (ii) the solutions of $x^2 - 16 = 0$; (iii) the set of points on a line.

[NCERT]

Solution:

- (i) 7 days — **finite**.
- (ii) $x = \pm 4$, i.e. $\{-4, 4\}$ — **finite**.
- (iii) A line has unlimited points — **infinite**.

Answer: finite, finite, infinite.

18. **[Finite / infinite]** Which is infinite? (i) $\{x \in \mathbb{N} : (x - 1)(x - 2) = 0\}$ (ii) $\{x \in \mathbb{N} : x \text{ is prime}\}$ (iii) $\{x \in \mathbb{N} : x^2 = 4\}$.

[NCERT]

Solution:

- (i) $\{1, 2\}$ — finite.
- (ii) primes 2, 3, 5, 7, 11, ... never end — **infinite**.
- (iii) $x^2 = 4, x \in \mathbb{N} \Rightarrow \{2\}$ — finite.

Answer: Only (ii) is infinite.

19. **[True / false]** True or false: the set of animals living on the Earth is finite; the set of circles passing through the origin is infinite.

[NCERT]

Solution: The number of animals on Earth, though huge, is a definite (countable) number — **finite**, so the first statement is *true*. Through the origin one can draw circles of every radius and centre — endlessly many — so the set of such circles is **infinite**, and the second statement is also *true*.

Answer: Both statements are **true**.

20. **[Equal sets]** Are $\{x : x \text{ is a letter of "FOLLOW"}\}$ and $\{y : y \text{ is a letter of "WOLF"}\}$ equal?

[NCERT]

Solution: List the distinct letters of each word (ignore repeats and order).

FOLLOW $\rightarrow \{F, O, L, W\}$

WOLF $\rightarrow \{W, O, L, F\}$.

Both contain exactly the same elements.

Answer: Yes, the sets are **equal**; each is $\{F, O, L, W\}$.

21. **[Equal sets]** Identify the pair of equal sets among $A = \{0\}$, $B = \{x : x > 15 \text{ and } x < 5\}$, $C = \{x : x - 5 = 0\}$, $D = \{x : x^2 = 25\}$, $E = \{x : x \text{ is a positive integral root of } x^2 - 2x - 15 = 0\}$. [NCERT]

Solution: Evaluate each set.

$$A = \{0\},$$

$$B = \emptyset \text{ (no } x \text{ is both } > 15 \text{ and } < 5),$$

$$C = \{5\},$$

$$D = \{x : x^2 = 25\} = \{-5, 5\}.$$

For E : $x^2 - 2x - 15 = 0 \Rightarrow (x - 5)(x + 3) = 0 \Rightarrow x = 5, -3$; the positive integral root is 5, so $E = \{5\}$.

Answer: $C = E = \{5\}$ is the only pair of equal sets.

22. **[Equal & equivalent]** From $A = \{0, a\}$, $B = \{1, 2, 3, 4\}$, $C = \{4, 8, 12\}$, $D = \{3, 1, 2, 4\}$, $E = \{1, 0\}$, $F = \{8, 4, 12\}$, $G = \{1, 5, 7, 11\}$, $H = \{a, b\}$, list (i) all pairs of equal sets, (ii) all groups of equivalent sets.

Solution: *Equal* = exactly the same elements; *equivalent* = the same *number* of elements.

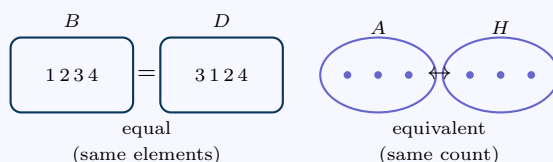
(i) **Equal sets:** $B = \{1, 2, 3, 4\}$ and $D = \{3, 1, 2, 4\}$ have identical elements, so $B = D$. Also $C = \{4, 8, 12\}$ and $F = \{8, 4, 12\}$ give $C = F$.

(ii) **Equivalent sets** (group by size):

2 elements: A, E, H ($n = 2$)

3 elements: C, F ($n = 3$)

4 elements: B, D, G ($n = 4$)



Answer: (i) $B = D$ and $C = F$. (ii) Equivalent groups: $\{A, E, H\}$, $\{C, F\}$, $\{B, D, G\}$.

23. **[Equal sets]** Are $A = \{1, 3, 5\}$ and $B = \{x : x \text{ is an odd natural number } < 6\}$ equal? [NCERT]

Solution: The odd natural numbers less than 6 are 1, 3, 5, so $B = \{1, 3, 5\} = A$.

Answer: Yes, $A = B$.

24. **[MCQ]** Which of the following is a **singleton** set?

- (A) $\{x : |x| = 5, x \in \mathbb{Z}\}$ (B) $\{x : x^2 = 4, x \in \mathbb{Z}\}$
 (C) $\{x : x^2 + x - 2 = 0, x \in \mathbb{N}\}$ (D) $\{x : x^2 = 9, x \in \mathbb{Z}\}$

Correct Answer: (C)

Solution: Count the elements of each.

$$(A) |x| = 5 \Rightarrow \{-5, 5\}$$

(2 elements)

$$(B) x^2 = 4 \Rightarrow \{-2, 2\}$$

(2 elements)

$$(C) x^2 + x - 2 = 0 \Rightarrow (x + 2)(x - 1) = 0 \Rightarrow x = -2, 1; x \in \mathbb{N} \Rightarrow \{1\}$$

(1 element)

$$(D) x^2 = 9 \Rightarrow \{-3, 3\}$$

(2 elements)

Only (C) has exactly one element.

Answer: (C).

25. [MCQ] Which of the following is a **null** set?

(A) $\{0\}$

(B) $\{\emptyset\}$

(C) $\{x : x^2 + 1 = 0, x \in \mathbb{R}\}$

(D) $\{x : x > 1\}$

Correct Answer: (C)

Solution: $\{0\}$ contains the element 0, and $\{\emptyset\}$ contains the element $\emptyset$ — both are *non-empty*. The equation $x^2 + 1 = 0$ has no real solution, so (C) = $\emptyset$. Set (D) is infinite.

Answer: (C).

26. [MCQ] Which of the following sets is **infinite**?

(A) the set of months of a year (B) $\{x \in \mathbb{N} : x < 100\}$

(C) the set of prime numbers less than 99 (D) $\{x \in \mathbb{R} : 0 < x < 1\}$

Correct Answer: (D)

Solution: (A) has 12 elements, (B) has 99, (C) is a finite list of primes. But between 0 and 1 there are uncountably many real numbers, so (D) is infinite.

Answer: (D).