Hi everyone,

The real learning in mathematics happens when you actively engage with a problem, explore different methods, and work through challenges. Therefore, we strongly encourage you to use this solution key responsibly.

Please attempt all the problems on your own first, giving them your best and most honest effort. These solutions are to help you get unstuck on a problem after you have already tried your best.

Your effort and dedication are the true keys to success.

Sets • Assignment 02 • Solutions

Topic: Subsets, Power Set & Intervals

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1. [Element vs Subset] State true or false: (i) $2 \in \{1,2,3\}$ (ii) $\{2\} \in \{1,2,3\}$ (iii) $\{2\} \subseteq \{1,2,3\}$ (iv) $\varnothing \subseteq \{1,2,3\}$ (v) $\varnothing \in \{1,2,3\}$ (vi) $\{1,2\} \subset \{1,2,3\}$.

Solution: Recall: "$\in$" compares an *object* with the elements of a set; "$\subseteq$" compares a *set* with another set (every element of the left set must lie in the right set). The elements of $\{1,2,3\}$ are the three numbers $1,2,3$.

(i) 2 is one of the listed elements. **True**. ✓

(ii) $\{2\}$ (a *set*) is not one of the elements $1,2,3$. **False**. ✗

(iii) Every element of $\{2\}$ (namely 2) lies in $\{1,2,3\}$. **True**. ✓

(iv) $\varnothing$ is a subset of every set. **True**. ✓

(v) $\varnothing$ is not listed as an element of $\{1,2,3\}$. **False**. ✗

(vi) $\{1,2\} \subseteq \{1,2,3\}$ and $\{1,2\} \neq \{1,2,3\}$, so it is a proper subset. **True**. ✓

Answer: (i) T (ii) F (iii) T (iv) T (v) F (vi) T.

2. [Element vs Subset] Let $A = \{1,2,\{3,4\},5\}$. State which are true: (i) $\{3,4\} \subset A$ (ii) $\{\{3,4\}\} \subset A$ (iii) $1 \subset A$ (iv) $\{1,2,3\} \subset A$ (v) $1 \in A$ (vi) $\{1,2,5\} \subset A$. *[NCERT]*

Solution: The four elements of A are: $1,\ 2,\ \{3,4\}$ (a single element which is itself a set), and 5.

(i) $\{3,4\} \subset A$ would require $3 \in A$ and $4 \in A$. But 3 and 4 are *not* elements of A (only the package $\{3,4\}$ is). **False**. ✗

(ii) $\{\{3,4\}\} \subset A$: its only element is $\{3,4\}$, which *is* an element of A. **True**. ✓

(iii) $1 \subset A$ is meaningless as written (1 is a number, not a set); it is **False**. ✗

(iv) $\{1,2,3\} \subset A$ needs $3 \in A$, but $3 \notin A$. **False**. ✗

(v) 1 is an element of A. **True**. ✓

(vi) $\{1,2,5\} \subset A$: each of $1,2,5$ is an element of A, and $\{1,2,5\} \neq A$. **True**. ✓

Answer: Only **(ii), (v) and (vi)** are true.

3. [Element vs Subset] For $A = \{1,2,\{3,4\},5\}$, state which are *incorrect*: (i) $\{3,4\} \in A$ (ii) $\{1,2,5\} \in A$ (iii) $\varnothing \in A$ (iv) $\varnothing \subset A$ (v) $\{\varnothing\} \subset A$. *[NCERT]*

Solution: Again $A = \{1,2,\{3,4\},5\}$ has elements $1,2,\{3,4\},5$. We flag the statements that are *wrong*.

(i) $\{3,4\} \in A$: yes, $\{3,4\}$ is literally one of the elements. **Correct**. ✓

(ii) $\{1,2,5\} \in A$: this would need the *set* $\{1,2,5\}$ to be an element of A, but it is not listed. (It is a subset, not an element.) **Incorrect**. ✗

(iii) $\varnothing \in A$: $\varnothing$ is not one of the listed elements of A. **Incorrect.** ✗

(iv) $\varnothing \subset A$: the empty set is a subset of every set. **Correct.** ✓

(v) $\{\varnothing\} \subset A$: this needs $\varnothing \in A$, which is false (see (iii)). **Incorrect.** ✗

Answer: The incorrect statements are **(ii), (iii) and (v)**.

4. [**Subsets**] Fill each blank with $\subseteq$ or $\nsubseteq$: (i) $\{3,4,5\}$ ___ $\{1,2,3,4,5\}$ (ii) $\{1,2,3\}$ ___ $\{2,3,4\}$ (iii) $\{x : x \text{ even natural}\}$ ___ $\mathbb{Z}$ (iv) $\{x : x \text{ a triangle}\}$ ___ $\{x : x \text{ a rectangle}\}$ (v) $\{x : x \text{ prime}\}$ ___ $\mathbb{N}$.

Solution: Check whether *every* element of the left set belongs to the right set.

(i) $3, 4, 5$ all lie in $\{1,2,3,4,5\}$, so $\subseteq$.

(ii) $1 \in \{1,2,3\}$ but $1 \notin \{2,3,4\}$, so $\nsubseteq$.

(iii) Every even natural number is an integer, so $\subseteq$.

(iv) A triangle is never a rectangle, so $\nsubseteq$.

(v) Every prime is a natural number, so $\subseteq$.

Answer: (i) $\subseteq$ (ii) $\nsubseteq$ (iii) $\subseteq$ (iv) $\nsubseteq$ (v) $\subseteq$.

5. [**Subsets**] Write down all the subsets of $\{a, b, c\}$.

Solution: A set with 3 elements has $2^3 = 8$ subsets. We list them by size:

$$\begin{aligned} \text{size } 0 &: \quad \varnothing \\ \text{size } 1 &: \quad \{a\},\ \{b\},\ \{c\} \\ \text{size } 2 &: \quad \{a,b\},\ \{a,c\},\ \{b,c\} \\ \text{size } 3 &: \quad \{a,b,c\} \end{aligned}$$

Answer: $\varnothing$, $\{a\}$, $\{b\}$, $\{c\}$, $\{a,b\}$, $\{a,c\}$, $\{b,c\}$, $\{a,b,c\}$ (8 subsets).

6. [**Proper subsets**] Write down all the *proper* subsets of $\{1, 2, 3\}$.

Solution: A proper subset is any subset *except* the set itself. There are $2^3 - 1 = 7$ of them (drop $\{1,2,3\}$ from the full list of 8):

$$\begin{aligned} \text{size } 0 &: \quad \varnothing \\ \text{size } 1 &: \quad \{1\},\ \{2\},\ \{3\} \\ \text{size } 2 &: \quad \{1,2\},\ \{1,3\},\ \{2,3\} \end{aligned}$$

Answer: $\varnothing$, $\{1\}$, $\{2\}$, $\{3\}$, $\{1,2\}$, $\{1,3\}$, $\{2,3\}$ (7 proper subsets).

7. [**Subsets**] List all the subsets of the set $\{1, \{2\}\}$. *[NCERT]*

Solution: The set has 2 elements: the number 1 and the *set* $\{2\}$. Hence $2^2 = 4$ subsets — keep $\{2\}$ together as a single element:

$$\begin{aligned} \text{size } 0 &: \quad \varnothing \\ \text{size } 1 &: \quad \{1\},\ \{\{2\}\} \\ \text{size } 2 &: \quad \{1, \{2\}\} \end{aligned}$$

Answer: $\varnothing$, $\{1\}$, $\{\{2\}\}$, $\{1, \{2\}\}$.

8. **[Power set]** Write the power set $P(\{7, 10, 11\})$ and verify it has $2^3 = 8$ elements.

Solution: The power set is the set of *all* subsets of $\{7, 10, 11\}$. List them by size:

$$\begin{aligned}
\text{size } 0: &\quad \varnothing \\
\text{size } 1: &\quad \{7\},\ \{10\},\ \{11\} \\
\text{size } 2: &\quad \{7, 10\},\ \{7, 11\},\ \{10, 11\} \\
\text{size } 3: &\quad \{7, 10, 11\}
\end{aligned}$$

Counting: $1 + 3 + 3 + 1 = 8 = 2^3$. ✓

Answer: $P(\{7, 10, 11\}) = \big\{\varnothing, \{7\}, \{10\}, \{11\}, \{7, 10\}, \{7, 11\}, \{10, 11\}, \{7, 10, 11\}\big\}$, and $n\big(P(\{7, 10, 11\})\big) = 8 = 2^3$.

9. **[Universal set]** Find the universal set with the least number of elements for $A = \{x : x^2 - 5x + 6 = 0\}$ and $B = \{x : x^2 - 3x + 2 = 0\}$. *[NCERT]*

Solution: First find A and B explicitly, then the smallest universal set is their union.

$$\begin{aligned}
A: &\quad x^2 - 5x + 6 = 0 \\
&(x - 2)(x - 3) = 0 \implies A = \{2, 3\} \\
B: &\quad x^2 - 3x + 2 = 0 \\
&(x - 1)(x - 2) = 0 \implies B = \{1, 2\}
\end{aligned}$$

$$A \cup B = \{2, 3\} \cup \{1, 2\} = \{1, 2, 3\}$$

Any universal set must contain every element of A and of B; the smallest such set is $A \cup B$.

Answer: Least universal set $U = \{1, 2, 3\}$.

10. **[Universal set]** Which can be a universal set for $A = \{1, 3, 5\}$, $B = \{2, 4, 6\}$, $C = \{0, 2, 4, 6, 8\}$: (i) $\{0, 1, \ldots, 6\}$ (ii) $\varnothing$ (iii) $\{0, 1, \ldots, 10\}$ (iv) $\{1, 2, \ldots, 8\}$? *[NCERT]*

Solution: A universal set must contain *all* elements of A, B and C. Collect them:

$$\begin{aligned}
A \cup B \cup C &= \{1, 3, 5\} \cup \{2, 4, 6\} \cup \{0, 2, 4, 6, 8\} \\
&= \{0, 1, 2, 3, 4, 5, 6, 8\}.
\end{aligned}$$

Now test each candidate:

(i) $\{0, 1, \ldots, 6\}$ misses 8. ✗

(ii) $\varnothing$ contains nothing. ✗

(iii) $\{0, 1, \ldots, 10\}$ contains $0, 1, 2, 3, 4, 5, 6, 8$ (and more). ✓

(iv) $\{1, 2, \ldots, 8\}$ misses 0. ✗

Answer: Only **(iii)** $\{0, 1, 2, \ldots, 10\}$ can serve as the universal set.

11. **[Power set]** If $P(A) = P(B)$, what is the relation between A and B? *[NCERT]*

Solution: Suppose $P(A) = P(B)$. Note that A is always its own largest subset, so $A \in P(A)$. Then:

$$\begin{aligned}
A \in P(A) = P(B) &\implies A \in P(B) \implies A \subseteq B, \\
B \in P(B) = P(A) &\implies B \in P(A) \implies B \subseteq A.
\end{aligned}$$

From $A \subseteq B$ and $B \subseteq A$ we conclude $A = B$.

Answer: $A = B$.

12. [Counting] The total number of subsets of a set containing 5 elements is ______. *[NCERT]*

Solution: A set with n elements has 2^n subsets. Here $n = 5$:

$$\begin{aligned} \text{number of subsets} &= 2^n \\ &= 2^5 \\ &= 32. \end{aligned}$$

Answer: 32.

13. [Counting] The number of proper subsets of $\{1, 2, 3\}$ is ______. *[NCERT]*

Solution: For n elements the number of proper subsets is $2^n - 1$ (all subsets except the set itself). Here $n = 3$:

$$\begin{aligned} \text{proper subsets} &= 2^n - 1 \\ &= 2^3 - 1 \\ &= 8 - 1 \\ &= 7. \end{aligned}$$

Answer: 7.

14. [Counting] The number of non-empty subsets of $\{1, 2, 3, 4\}$ is ______. *[NCERT]*

Solution: For n elements the number of non-empty subsets is $2^n - 1$ (all subsets except $\varnothing$). Here $n = 4$:

$$\begin{aligned} \text{non-empty subsets} &= 2^n - 1 \\ &= 2^4 - 1 \\ &= 16 - 1 \\ &= 15. \end{aligned}$$

Answer: 15.

15. [Counting] Two finite sets have m and n elements. The number of subsets of the first is 48 more than that of the second. Find m and n.

Solution: The first set has 2^m subsets and the second has 2^n subsets. The condition is

$$\begin{aligned} 2^m - 2^n &= 48 \\ 2^n\left(2^{m-n} - 1\right) &= 48 = 2^4 \cdot 3. \end{aligned}$$

Since $2^{m-n} - 1$ is odd, the factor of 2^4 must come entirely from 2^n, so $2^n = 16$, i.e. $n = 4$, and then $2^{m-n} - 1 = 3 \Rightarrow 2^{m-n} = 4 \Rightarrow m - n = 2$, giving $m = 6$. Check: $2^6 - 2^4 = 64 - 16 = 48$. ✓

Answer: $m = 6,\ n = 4$.

16. [Power set] Find $n\big\{P(P(P(P(\varnothing))))\big\}$. *[NCERT]*

Solution: Build up one power set at a time, tracking the number of elements (recall $n(P(S)) = 2^{\,n(S)}$).

$$\begin{aligned} \varnothing: &\qquad n = 0 \\ P(\varnothing) = \{\varnothing\}: &\qquad n = 2^0 = 1 \\ P(P(\varnothing)): &\qquad n = 2^1 = 2 \\ P(P(P(\varnothing))): &\qquad n = 2^2 = 4 \end{aligned}$$

$$P(P(P(P(\varnothing)))): \qquad n = 2^4 = 16.$$

Answer: $n\{P(P(P(P(\varnothing))))\} = 16.$

17. [Counting] If a set A has exactly 128 subsets, the number of elements in A is ______.

Solution: If A has n elements then it has 2^n subsets. Set this equal to 128:

$$\begin{aligned} 2^n &= 128 \\ 2^n &= 2^7 \\ n &= 7. \end{aligned}$$

Answer: $n = 7$.

18. [Counting] The number of subsets of $\{1, 2, 3, 4, 5\}$ that contain the element 3 is ______.

Solution: Force 3 into the subset. The remaining 4 elements $\{1, 2, 4, 5\}$ may be freely included or excluded, so the count is 2^4:

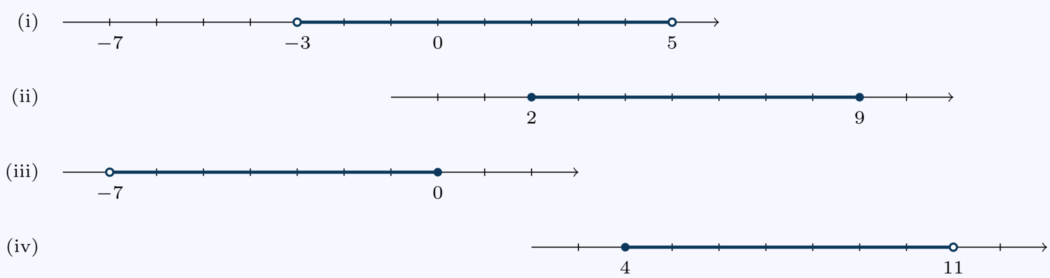
$$\begin{aligned} \text{subsets containing } 3 &= 2^{\,5-1} \\ &= 2^4 \\ &= 16. \end{aligned}$$

Answer: 16.

19. [Intervals] Write as intervals: (i) $\{x \in \mathbb{R} : -3 < x < 5\}$ (ii) $\{x \in \mathbb{R} : 2 \le x \le 9\}$ (iii) $\{x \in \mathbb{R} : -7 < x \le 0\}$ (iv) $\{x \in \mathbb{R} : 4 \le x < 11\}$.

Solution: A strict inequality ($<$) gives an *open* end (round bracket, open circle); a non-strict inequality ($\le$) gives a *closed* end (square bracket, filled dot).

$$\begin{aligned} \text{(i)} \quad -3 < x < 5 &\implies (-3,\ 5) \\ \text{(ii)} \quad 2 \le x \le 9 &\implies [2,\ 9] \\ \text{(iii)} \quad -7 < x \le 0 &\implies (-7,\ 0] \\ \text{(iv)} \quad 4 \le x < 11 &\implies [4,\ 11) \end{aligned}$$

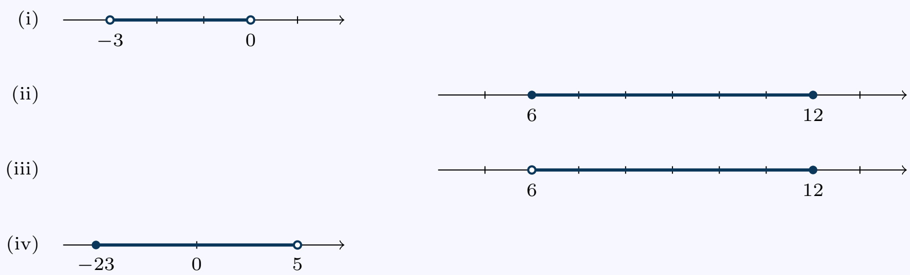


Answer: (i) $(-3, 5)$ (ii) $[2, 9]$ (iii) $(-7, 0]$ (iv) $[4, 11)$.

20. [Intervals] Write in set-builder form: (i) $(-3, 0)$ (ii) $[6, 12]$ (iii) $(6, 12]$ (iv) $[-23, 5)$. *[NCERT]*

Solution: A round bracket means the endpoint is excluded (strict $<$); a square bracket means it is included (non-strict $\leq$).

$$\begin{aligned} &\text{(i)} \quad (-3,0) \implies \{x \in \mathbb{R} : -3 < x < 0\} \\ &\text{(ii)} \quad [6,12] \implies \{x \in \mathbb{R} : 6 \leq x \leq 12\} \\ &\text{(iii)} \quad (6,12] \implies \{x \in \mathbb{R} : 6 < x \leq 12\} \\ &\text{(iv)} \quad [-23,5) \implies \{x \in \mathbb{R} : -23 \leq x < 5\} \end{aligned}$$

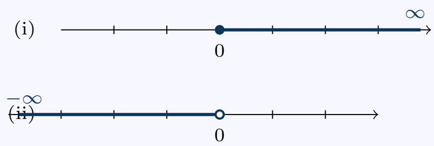


Answer: (i) $\{x : -3 < x < 0\}$ (ii) $\{x : 6 \leq x \leq 12\}$ (iii) $\{x : 6 < x \leq 12\}$ (iv) $\{x : -23 \leq x < 5\}$.

21. **[Intervals]** Write the interval representing (i) all non-negative reals, (ii) all negative reals. *[NCERT]*

Solution: "Non-negative" means $x \geq 0$ (includes 0); "negative" means $x < 0$ (excludes 0). The ∞ symbols always take a round bracket.

$$\begin{aligned} &\text{(i)} \quad \{x \in \mathbb{R} : x \geq 0\} \implies [0,\infty) \\ &\text{(ii)} \quad \{x \in \mathbb{R} : x < 0\} \implies (-\infty,0) \end{aligned}$$



Answer: (i) $[0,\infty)$ (ii) $(-\infty,0)$.

22. **[MCQ]** The number of proper subsets of a set with n elements is

(A) 2^n (B) $2^n - 1$ (C) $2^n - 2$ (D) n^2

Correct Answer: (B)

Solution: A set with n elements has 2^n subsets in total. A *proper* subset is any subset other than the set itself, so we remove exactly one:

$$\begin{aligned} \text{proper subsets} &= (\text{all subsets}) - 1 \\ &= 2^n - 1. \end{aligned}$$

Answer: (B) $2^n - 1$.

23. **[MCQ]** If $A = \varnothing$, then $P(A)$ is

(A) $\varnothing$ (B) $\{\varnothing\}$ (C) $\{0\}$ (D) $\{\varnothing, \{\varnothing\}\}$

Correct Answer: (B)

Solution: The power set is the set of all subsets. The empty set has exactly one subset — itself:

$$\begin{aligned} A &= \varnothing \\ \text{subsets of } A: &\ \ \varnothing \\ P(A) &= \{\varnothing\}. \end{aligned}$$

Note $P(\varnothing) = \{\varnothing\}$ is *not* empty: it contains one element, namely $\varnothing$. Indeed $n\big(P(\varnothing)\big) = 2^0 = 1$.

Answer: (B) $\{\varnothing\}$.

24. **[MCQ]** $n\big(P(P(P(\varnothing)))\big)$ equals

(A) 2 (B) 4 (C) 8 (D) 16

Correct Answer: (B)

Solution: Apply $n(P(S)) = 2^{\,n(S)}$ three times, starting from $n(\varnothing) = 0$:

$$\begin{aligned} n\big(P(\varnothing)\big) &= 2^0 = 1 \\ n\big(P(P(\varnothing))\big) &= 2^1 = 2 \\ n\big(P(P(P(\varnothing)))\big) &= 2^2 = 4. \end{aligned}$$

Answer: (B) 4.

25. **[MCQ]** If $X = \{8^n - 7n - 1 : n \in \mathbb{N}\}$ and $Y = \{49(n-1) : n \in \mathbb{N}\}$, then

(A) $X \subseteq Y$ (B) $Y \subseteq X$
(C) $X = Y$ (D) None of these

Correct Answer: (A)

Solution: The set $Y = \{49(n-1) : n \in \mathbb{N}\} = \{0, 49, 98, 147, \ldots\}$ is exactly the set of *all* non-negative multiples of 49. We show every element of X is such a multiple. We can express 8^n using the sum of a geometric series:

$$\frac{8^n - 1}{8 - 1} = 1 + 8 + 8^2 + \cdots + 8^{n-1} \implies 8^n - 1 = 7\sum_{k=0}^{n-1} 8^k.$$

Substituting this into our expression:

$$\begin{aligned} 8^n - 7n - 1 &= (8^n - 1) - 7n \\ &= 7\sum_{k=0}^{n-1} 8^k - 7n \\ &= 7\sum_{k=0}^{n-1} (8^k - 1) \\ &= 7\sum_{k=1}^{n-1} (8^k - 1). \end{aligned}$$

Since $8^k - 1$ is divisible by $8 - 1 = 7$ for any integer $k \geq 1$, we can factor out another 7:

$$8^k - 1 = 7(8^{k-1} + 8^{k-2} + \cdots + 1).$$

Therefore, we get:

$$\begin{aligned} 8^n - 7n - 1 &= 7\sum_{k=1}^{n-1} 7(8^{k-1} + 8^{k-2} + \cdots + 1) \\ &= 49\sum_{k=1}^{n-1} (8^{k-1} + 8^{k-2} + \cdots + 1) \end{aligned}$$

$$= 49 \cdot (\text{an integer}).$$

Hence every element of X is a (non-negative) multiple of 49, i.e. $X \subseteq Y$. (In fact X is a *proper* subset of Y: for instance $49 = 49(2-1) \in Y$, but no n gives $8^n - 7n - 1 = 49$, so $X \neq Y$.)

Answer: (A) $X \subseteq Y$.

26. [MCQ] Which of the following is a subset of *every* set?

(A) $\{0\}$
(B) $\varnothing$
(C) $\{\varnothing\}$
(D) the universal set

Correct Answer: (B)

Solution: The empty set $\varnothing$ is a subset of every set (vacuously: there is no element of $\varnothing$ that could fail to lie in the other set). The other choices are non-empty sets and so are *not* subsets of, e.g., $\varnothing$ itself:

$$\begin{aligned} &\{0\} \text{ contains } 0 && \text{(not a subset of a set lacking 0)}, \\ &\{\varnothing\} \text{ contains } \varnothing && \text{(not a subset of a set lacking } \varnothing), \\ &\text{universal set contains everything} && \text{(not a subset of a smaller set)}. \end{aligned}$$

Answer: (B) $\varnothing$.