Hi everyone,

The real learning in mathematics happens when you actively engage with a problem, explore different methods, and work through challenges. Therefore, we strongly encourage you to use this solution key responsibly.

Please attempt all the problems on your own first, giving them your best and most honest effort. These solutions are to help you get unstuck on a problem after you have already tried your best.

Your effort and dedication are the true keys to success.

Sets • Assignment 03 • Solutions

Topic: Operations & Algebra of Sets

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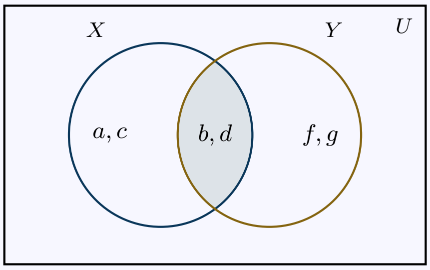
1. [Set operations] If $X = \{a, b, c, d\}$ and $Y = \{f, b, d, g\}$, find $X - Y$, $Y - X$, $X \cap Y$ and $X \cup Y$. *[NCERT]*

Solution: Read off each operation directly from the two lists.

$$X - Y = \{\, x \in X : x \notin Y \,\} = \{a, c\}$$
$$Y - X = \{\, y \in Y : y \notin X \,\} = \{f, g\}$$
$$X \cap Y = \{\, \text{elements in both} \,\} = \{b, d\}$$
$$X \cup Y = \{\, \text{all elements together} \,\} = \{a, b, c, d, f, g\}$$



The blue region is $X - Y$, the gold region is $Y - X$, and the overlap is $X \cap Y$.

Answer: $X - Y = \{a, c\}$, $Y - X = \{f, g\}$, $X \cap Y = \{b, d\}$, $X \cup Y = \{a, b, c, d, f, g\}$.

2. [De Morgan's law] Let $U = \{1, 2, \ldots, 9\}$, $A = \{2, 4, 6, 8\}$, $B = \{2, 3, 5, 7\}$. Find A', B' and hence verify $(A \cup B)' = A' \cap B'$. *[NCERT]*

Solution: The complement of a set is everything in U not in it.

$$A' = U - A = \{1, 3, 5, 7, 9\}$$
$$B' = U - B = \{1, 4, 6, 8, 9\}$$

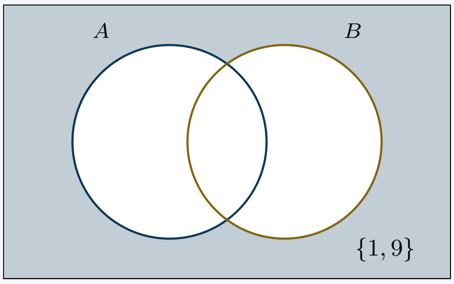
Now compute the left side $(A \cup B)'$:

$$A \cup B = \{2, 3, 4, 5, 6, 7, 8\}$$
$$(A \cup B)' = U - (A \cup B) = \{1, 9\}$$

And the right side $A' \cap B'$:

$$A' \cap B' = \{1, 3, 5, 7, 9\} \cap \{1, 4, 6, 8, 9\} = \{1, 9\}$$

Both sides equal $\{1, 9\}$, so the identity is verified. The Venn picture shows $(A \cup B)'$ as the region *outside* both circles.



Answer: $A' = \{1, 3, 5, 7, 9\}$, $B' = \{1, 4, 6, 8, 9\}$, and $(A \cup B)' = A' \cap B' = \{1, 9\}$ (verified).

3. [Region-method proof] Prove that $A - (B \cup C) = (A - B) \cap (A - C)$. *[NCERT]*

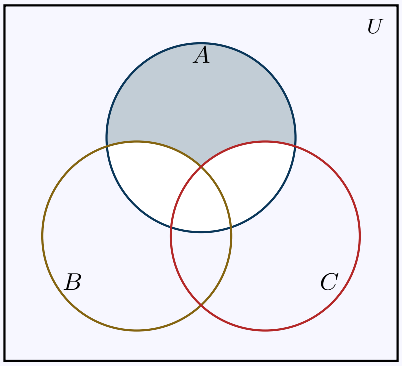
Solution: We argue by the region method, showing both sides describe *the part of A lying outside B and outside C.*
Left side. An element x lies in $A - (B \cup C)$ exactly when

$$\begin{aligned} x \in A - (B \cup C) &\iff x \in A \text{ and } x \notin (B \cup C) \\ &\iff x \in A \text{ and } \big(x \notin B \text{ and } x \notin C\big). \end{aligned}$$

Right side. An element x lies in $(A - B) \cap (A - C)$ exactly when

$$\begin{aligned} x \in (A - B) \cap (A - C) &\iff (x \in A \text{ and } x \notin B) \text{ and } (x \in A \text{ and } x \notin C) \\ &\iff x \in A \text{ and } \big(x \notin B \text{ and } x \notin C\big). \end{aligned}$$

The two conditions are identical, so the sets are equal. In the Venn diagram below, the shaded region is the slice of A that avoids both B and C — this is simultaneously $A - (B \cup C)$ (start with A, remove everything in B or C) and $(A - B) \cap (A - C)$ (the part of A outside B, intersected with the part of A outside C).



Answer: Both sides equal the elements of A that lie outside B *and* outside C; hence $A - (B \cup C) = (A - B) \cap (A - C)$. ■

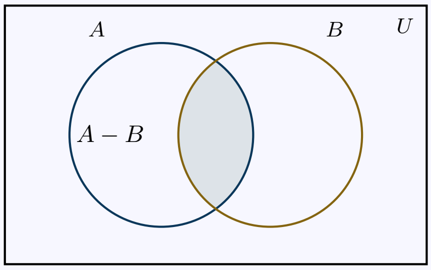
4. [Algebra of sets] For any two sets, $A - B$ equals

(A) $A \cap B$ (B) $A' \cap B$ (C) $A \cap B'$ (D) $A \cup B'$

Correct Answer: (C)

Solution: "$x \in A$ but $x \notin B$" is the same as "$x \in A$ and $x \in B'$".

$$\begin{aligned} A - B &= \{\, x : x \in A \text{ and } x \notin B \,\} \\ &= \{\, x : x \in A \text{ and } x \in B' \,\} \\ &= A \cap B' \end{aligned}$$



The blue-only region (in A, outside B) is exactly $A \cap B'$.

Answer: (C) $A \cap B'$.

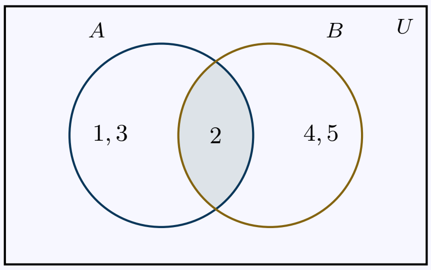
5. [**Symmetric difference**] The symmetric difference of $A = \{1, 2, 3\}$ and $B = \{2, 4, 5\}$ is

(A) $\{1, 2\}$ (B) $\{1, 3, 4, 5\}$ (C) $\{3, 4\}$ (D) $\{1, 2, 3, 4, 5\}$

Correct Answer: (B)

Solution: The symmetric difference collects elements in exactly one of the sets.

$$A - B = \{1, 3\}$$
$$B - A = \{4, 5\}$$
$$A \triangle B = (A - B) \cup (B - A) = \{1, 3\} \cup \{4, 5\} = \{1, 3, 4, 5\}$$



The common element 2 is dropped; what remains is $\{1, 3, 4, 5\}$.

Answer: (B) $\{1, 3, 4, 5\}$.

6. [**Symmetric difference**] $A \triangle B$ is **not** equal to

(A) $(A - B) \cup (B - A)$ (B) $(A \cup B) - (A \cap B)$
(C) $(A \cap B) - (A \cup B)$ (D) $(B - A) \cup (A - B)$

Correct Answer: (C)

Solution: Options (A), (B), (D) are all standard equivalent forms of $A \triangle B$. Check (C):

$$A \cap B \subseteq A \cup B$$
$$(A \cap B) - (A \cup B) = \varnothing$$

Since $A \cap B$ is a subset of $A \cup B$, subtracting the larger set from the smaller leaves nothing. So (C) gives $\varnothing$, which is *not* equal to $A \triangle B$ in general.

Answer: (C) $(A \cap B) - (A \cup B) = \varnothing$, so it is not equal to $A \triangle B$.

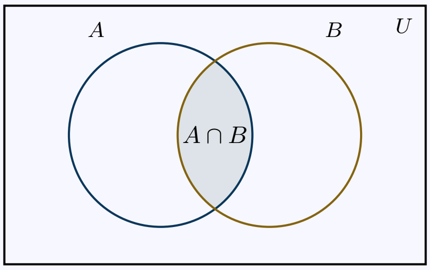
7. [**De Morgan's law**] $(A' \cup B')'$ equals

(A) $A \cup B$ (B) $A \cap B$ (C) $A' \cap B'$ (D) $A - B$

Correct Answer: (B)

Solution: Apply De Morgan's law to the union, then use $(A')' = A$.

$$\begin{aligned}(A' \cup B')' &= (A')' \cap (B')' \\ &= A \cap B\end{aligned}$$



The doubly-complemented expression collapses back to the overlap $A \cap B$.

Answer: (B) $A \cap B$.

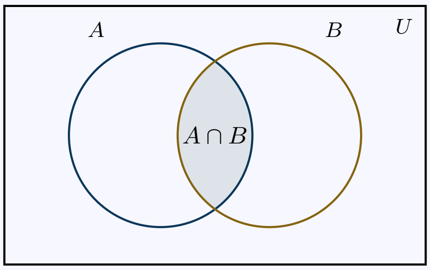
8. [Algebra of sets] $A - (A - B)$ equals

(A) $\varnothing$ (B) B (C) $A \cap B$ (D) $A \cup B$

Correct Answer: (C)

Solution: Rewrite each difference as an intersection with a complement.

$$\begin{aligned}A - B &= A \cap B' \\ A - (A - B) &= A \cap (A \cap B')' \\ &= A \cap (A' \cup B) && \text{(De Morgan)} \\ &= (A \cap A') \cup (A \cap B) && \text{(distributive)} \\ &= \varnothing \cup (A \cap B) \\ &= A \cap B\end{aligned}$$



Removing from A "the part of A not in B" leaves exactly $A \cap B$.

Answer: (C) $A \cap B$.

9. [Algebra of sets] $A \cap (A \cup B)'$ equals

(A) A (B) $A \cap B$ (C) $\varnothing$ (D) B

Correct Answer: (C)

Solution: Expand the complement with De Morgan, then simplify.

$$\begin{aligned}A \cap (A \cup B)' &= A \cap (A' \cap B') && \text{(De Morgan)} \\ &= (A \cap A') \cap B' \\ &= \varnothing \cap B' \\ &= \varnothing\end{aligned}$$

Since $(A \cup B)'$ lies entirely outside A, it cannot meet A at all.

Answer: (C) $\varnothing$.

10. [**Algebra of sets**] If A is the set of multiples of 3 and B the set of multiples of 5, then $A - B$ equals

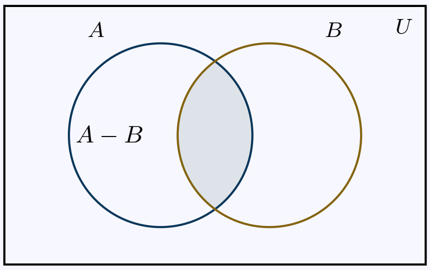
(A) $A \cap B$ (B) $A \cup B$ (C) $A \cap B'$ (D) $B - A$

Correct Answer: (C)

Solution: "Multiples of 3 that are *not* multiples of 5" is precisely A with B removed.

$$\begin{aligned} A - B &= \{\, x : x \in A \text{ and } x \notin B \,\} \\ &= A \cap B' \end{aligned}$$

For instance $3, 6, 9, 12$ stay, while $15, 30$ (also multiples of 5) are excluded.



Answer: (C) $A \cap B'$.

11. [**Subsets**] If $A \cup B = B$, then

(A) $A \subseteq B$ (B) $B \subseteq A$ (C) $A = \varnothing$ (D) $A = B$

Correct Answer: (A)

Solution: Use the fact that $A \subseteq A \cup B$ always.

$$\begin{aligned} A &\subseteq A \cup B && \text{(always true)} \\ A \cup B &= B && \text{(given)} \\ \therefore\ A &\subseteq B \end{aligned}$$

Adding B to A changes nothing only when everything in A was already in B.

Answer: (A) $A \subseteq B$.

12. [**Counting**] If A and B are disjoint, then $n(A \cup B)$ equals

(A) $n(A) + n(B)$ (B) $n(A) + n(B) - n(A \cap B)$
(C) $n(A) \cdot n(B)$ (D) 0

Correct Answer: (A)

Solution: The inclusion–exclusion principle reads

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

Disjoint sets share no elements, so $A \cap B = \varnothing$ and $n(A \cap B) = 0$:

$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - 0 \\ &= n(A) + n(B) \end{aligned}$$

Answer: (A) $n(A) + n(B)$.

13. [Sets of figures] Let F_1 = parallelograms, F_2 = rectangles, F_3 = rhombuses, F_4 = squares (in a plane). Then $F_2 \cap F_3$ equals

(A) F_1 (B) F_4 (C) $\varnothing$ (D) $F_2 \cup F_3$

Correct Answer: (B)

Solution: A figure that is *both* a rectangle and a rhombus has all angles equal to 90° (rectangle) *and* all sides equal (rhombus) — that is exactly a square.

$$\begin{aligned} F_2 \cap F_3 &= \{\text{rectangles}\} \cap \{\text{rhombuses}\} \\ &= \{\text{four equal sides and four right angles}\} \\ &= \{\text{squares}\} = F_4 \end{aligned}$$

Answer: (B) F_4 (squares).

14. [Match the columns] Match each expression in Column I with its simplest form in Column II.

Column I	**Column II**
(i) $A \cap (A \cup B)$	(P) $A \cap B$
(ii) $(A \cup B)'$	(Q) $A' \cap B'$
(iii) $A - (A - B)$	(R) $A \triangle B$
(iv) $(A - B) \cup (B - A)$	(S) A

Correct Answer: (i)→S, (ii)→Q, (iii)→P, (iv)→R

Solution: Simplify each expression in turn.

$$\begin{aligned} &\text{(i)} \quad A \cap (A \cup B) = A && \text{(absorption)} && \Rightarrow \text{S} \\ &\text{(ii)} \quad (A \cup B)' = A' \cap B' && \text{(De Morgan)} && \Rightarrow \text{Q} \\ &\text{(iii)} \quad A - (A - B) = A \cap (A' \cup B) = A \cap B && && \Rightarrow \text{P} \\ &\text{(iv)} \quad (A - B) \cup (B - A) = A \triangle B && \text{(definition)} && \Rightarrow \text{R} \end{aligned}$$

Answer: (i)→S, (ii)→Q, (iii)→P, (iv)→R.

15. [Distributive law] $A \cup (B \cap C)$ equals

(A) $(A \cup B) \cap (A \cup C)$ (B) $(A \cap B) \cup (A \cap C)$
(C) $A \cap B \cap C$ (D) $A \cup B \cup C$

Correct Answer: (A)

Solution: This is the distributive law of union over intersection.

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

(Note that option (B) is the *other* distributive law, $A \cap (B \cup C)$, so it does not apply here.)

Answer: (A) $(A \cup B) \cap (A \cup C)$.

16. [**Algebra of sets**] $(A \cap B) \cup (A - B)$ equals

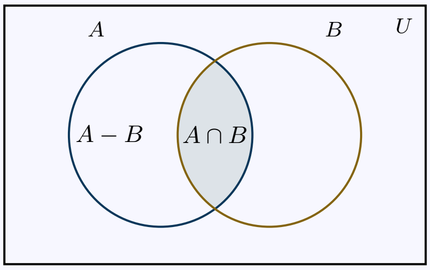
(A) A (B) B (C) $A \cap B$ (D) $A \cup B$

Correct Answer: (A)

Solution: Write $A - B = A \cap B'$ and factor out A.

$$\begin{aligned}(A \cap B) \cup (A - B) &= (A \cap B) \cup (A \cap B') \\ &= A \cap (B \cup B') && \text{(distributive)} \\ &= A \cap U \\ &= A\end{aligned}$$

The part of A inside B together with the part of A outside B rebuilds all of A.



Answer: (A) A.

17. [**Subsets**] If $A \subseteq B$, then $\big(A \cap B,\ A \cup B\big)$ equals

(A) $(A,\, B)$ (B) $(B,\, A)$ (C) $(A,\, A)$ (D) $(B,\, B)$

Correct Answer: (A)

Solution: When $A \subseteq B$, the smaller set is the overlap and the larger set is the union.

$$A \subseteq B \Rightarrow A \cap B = A$$
$$A \subseteq B \Rightarrow A \cup B = B$$

So the ordered pair is $(A,\, B)$.

Answer: (A) $(A,\, B)$.

18. [**Symmetric difference**] If $A \triangle B = A \triangle C$, then

(A) $A = B$ (B) $B = C$ (C) $A = C$ (D) $A = \varnothing$

Correct Answer: (B)

Solution: Symmetric difference behaves like "addition mod 2": it is associative, and every set is its own inverse, since $A \triangle A = \varnothing$ and $A \triangle \varnothing = A$. "$\triangle$" with A on the left of the given equation:

$$\begin{aligned}A \triangle B &= A \triangle C \\ A \triangle (A \triangle B) &= A \triangle (A \triangle C) \\ (A \triangle A) \triangle B &= (A \triangle A) \triangle C \\ \varnothing \triangle B &= \varnothing \triangle C \\ B &= C\end{aligned}$$

Answer: (B) $B = C$.

19. [**Algebra of sets**] For subsets A, B, C of U, the set $(A' \cap B' \cap C) \cup (B \cap C) \cup (A \cap C)$ equals

(A) A (B) B (C) C (D) $A \cap B \cap C$

Correct Answer: (C)

Solution: Every term contains C, so factor C out, then simplify the bracket.

$$\begin{aligned}
(A' \cap B' \cap C) \cup (B \cap C) \cup (A \cap C) &= C \cap \big[(A' \cap B') \cup B \cup A\big] \\
&= C \cap \big[(A \cup B)' \cup (A \cup B)\big] && \text{(De Morgan on } A' \cap B'\text{)} \\
&= C \cap U && \text{(set} \cup \text{its complement)} \\
&= C
\end{aligned}$$

The bracket is a set together with its own complement, which fills all of U.

Answer: (C) C.

20. [**Distributive law**] $A \cap (B \triangle C)$ equals

(A) $(A \cap B) \cup (A \cap C)$ (B) $(A \cap B) \triangle (A \cap C)$
(C) $(A \cup B) \triangle (A \cup C)$ (D) $A \cap B \cap C$

Correct Answer: (B)

Solution: Intersection distributes over symmetric difference. Writing $B \triangle C = (B - C) \cup (C - B)$:

$$\begin{aligned}
A \cap (B \triangle C) &= A \cap \big[(B \cap C') \cup (C \cap B')\big] \\
&= (A \cap B \cap C') \cup (A \cap C \cap B') \\
&= \big[(A \cap B) - (A \cap C)\big] \cup \big[(A \cap C) - (A \cap B)\big] \\
&= (A \cap B) \triangle (A \cap C)
\end{aligned}$$

Answer: (B) $(A \cap B) \triangle (A \cap C)$.

21. [**Subsets**] If $X = \{4^n - 3n - 1 : n \in \mathbb{N}\}$ and $Y = \{9(n-1) : n \in \mathbb{N}\}$, then $X \cup Y$ equals

(A) X (B) Y (C) $\varnothing$ (D) $\mathbb{N}$

Correct Answer: (B)

Solution: $Y = \{0, 9, 18, 27, \dots\}$ is the set of *all* non-negative multiples of 9. We show each element of X is a multiple of 9, i.e. $X \subseteq Y$. We can express 4^n using the sum of a geometric series:

$$\frac{4^n - 1}{4 - 1} = 1 + 4 + 4^2 + \cdots + 4^{n-1} \implies 4^n - 1 = 3\sum_{k=0}^{n-1} 4^k.$$

Substituting this into our expression:

$$\begin{aligned}
4^n - 3n - 1 &= (4^n - 1) - 3n \\
&= 3\sum_{k=0}^{n-1} 4^k - 3n \\
&= 3\sum_{k=0}^{n-1} (4^k - 1) \\
&= 3\sum_{k=1}^{n-1} (4^k - 1).
\end{aligned}$$

Since $4^k - 1$ is divisible by $4 - 1 = 3$ for any integer $k \geq 1$, we can factor out another 3:

$$4^k - 1 = 3(4^{k-1} + 4^{k-2} + \cdots + 1).$$

Therefore, we get:

$$\begin{aligned}4^n - 3n - 1 &= 3\sum_{k=1}^{n-1} 3(4^{k-1} + 4^{k-2} + \cdots + 1)\\ &= 9\sum_{k=1}^{n-1}(4^{k-1} + 4^{k-2} + \cdots + 1)\\ &= 9 \cdot (\text{integer}).\end{aligned}$$

So every element of X is a multiple of 9, giving $X \subseteq Y$. Therefore

$$X \cup Y = Y.$$

Answer: (B) Y (since $4^n - 3n - 1$ is always a multiple of 9, so $X \subseteq Y$).

22. [**Algebra of sets**] The set $\big(A \cap (B \cup C)'\big) \cup \big(B \cap (A \cup C)'\big) \cup \big(C \cap (A \cup B)'\big)$ consists of the elements lying in

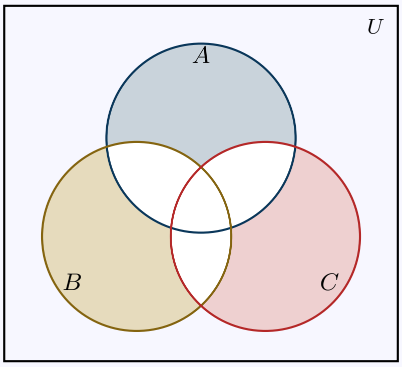
(A) all three of A, B, C
(B) exactly one of A, B, C
(C) exactly two of A, B, C
(D) at least one of A, B, C

Correct Answer: (B)

Solution: Examine a single term. Using De Morgan, $(B \cup C)' = B' \cap C'$:

$$\begin{aligned}A \cap (B \cup C)' &= A \cap B' \cap C'\\ &= \{\, x : x \in A,\ x \notin B,\ x \notin C \,\}.\end{aligned}$$

This is the part of A lying in *neither* B nor C — i.e. elements in A **only**. By symmetry the other two terms describe elements in B only and in C only. Their union is therefore the elements belonging to **exactly one** of the three sets.



The three shaded "petals" are exactly the elements lying in just one set.

Answer: (B) exactly one of A, B, C.