Hi everyone,

The real learning in mathematics happens when you actively engage with a problem, explore different methods, and work through challenges. Therefore, we strongly encourage you to use this solution key responsibly.

Please attempt all the problems on your own first, giving them your best and most honest effort. These solutions are to help you get unstuck on a problem after you have already tried your best.

Your effort and dedication are the true keys to success.

Sets • Assignment 04 • Solutions

Topic: Venn Diagrams & Cardinality (Two Sets)

Sub: Mathematics **11th • JEE** Chetan Sir

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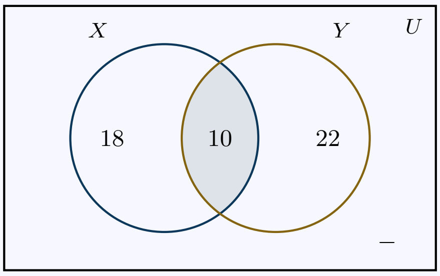
1. **[Using the formula]** If $n(X) = 28$, $n(Y) = 32$ and $n(X \cup Y) = 50$, find $n(X \cap Y)$. *[NCERT]*

Solution: Use the inclusion–exclusion identity and solve for the overlap.

$$\begin{aligned} n(X \cup Y) &= n(X) + n(Y) - n(X \cap Y) \\ 50 &= 28 + 32 - n(X \cap Y) \\ n(X \cap Y) &= 60 - 50 \\ n(X \cap Y) &= 10. \end{aligned}$$



Answer: $n(X \cap Y) = 10$.

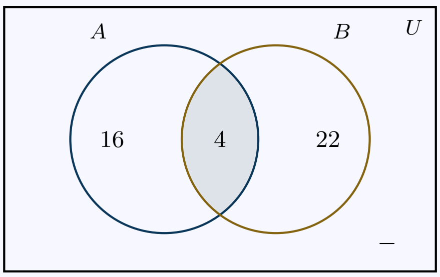
2. **[Using the formula]** Let $n(A) = 20$, $n(A \cup B) = 42$ and $n(A \cap B) = 4$. Find $n(B)$, $n(A - B)$ and $n(B - A)$.

Solution: First recover $n(B)$ from inclusion–exclusion, then peel off the "only" parts.

$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ 42 &= 20 + n(B) - 4 \\ n(B) &= 42 - 16 \\ n(B) &= 26. \end{aligned}$$

Now the set differences (the "only" regions):

$$\begin{aligned} n(A - B) &= n(A) - n(A \cap B) = 20 - 4 = 16, \\ n(B - A) &= n(B) - n(A \cap B) = 26 - 4 = 22. \end{aligned}$$

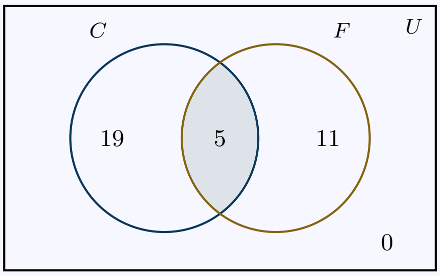


Answer: $n(B) = 26, \quad n(A - B) = 16, \quad n(B - A) = 22.$

3. [**Word problem**] In a class of 35 students, 24 like cricket and 16 like football; each likes at least one. How many like both? *[NCERT]*

Solution: "Each likes at least one" means the union covers all 35 students, so $n(C \cup F) = 35$.

$$\begin{aligned} n(C \cup F) &= n(C) + n(F) - n(C \cap F) \\ 35 &= 24 + 16 - n(C \cap F) \\ n(C \cap F) &= 40 - 35 \\ n(C \cap F) &= 5. \end{aligned}$$



Answer: **5** students like both cricket and football.

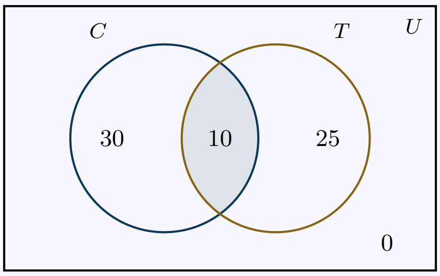
4. [**Word problem**] In a group of 65 people, 40 like cricket and 10 like both cricket and tennis; each likes at least one. Find (i) those who like tennis only, (ii) those who like tennis. *[NCERT]*

Solution: Let C = cricket, T = tennis. Given $n(C \cup T) = 65$, $n(C) = 40$, $n(C \cap T) = 10$.
(i) Tennis only. Everyone is in the union, so tennis-only fills the part outside cricket:

$$\begin{aligned} n(T \text{ only}) &= n(C \cup T) - n(C) \\ &= 65 - 40 \\ &= 25. \end{aligned}$$

(ii) Those who like tennis. Add back the overlap:

$$\begin{aligned} n(T) &= n(T \text{ only}) + n(C \cap T) \\ &= 25 + 10 \\ &= 35. \end{aligned}$$



Answer: (i) Tennis only = **25**. (ii) Tennis = **35**.

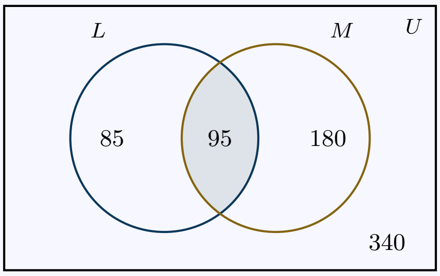
5. [**Word problem**] In a survey of 700 college students, 180 drink Limca, 275 drink Mirinda and 95 drink both. Find (i) how many drink Limca only, (ii) how many drink neither.

Solution: Let L = Limca, M = Mirinda, with $n(U) = 700$, $n(L) = 180$, $n(M) = 275$, $n(L \cap M) = 95$.
(i) Limca only.

$$\begin{aligned} n(L \text{ only}) &= n(L) - n(L \cap M) \\ &= 180 - 95 \\ &= 85. \end{aligned}$$

(ii) Neither. First find the union, then subtract from the universe:

$$\begin{aligned} n(L \cup M) &= 180 + 275 - 95 = 360, \\ n(\text{neither}) &= n(U) - n(L \cup M) \\ &= 700 - 360 \\ &= 340. \end{aligned}$$



Answer: (i) Limca only = **85**. (ii) Neither = **340**.

6. [Word problem] Of 200 individuals with a skin disorder, 120 were exposed to chemical C_1, 50 to C_2 and 30 to both. Find the number exposed to (i) C_1 but not C_2, (ii) C_2 but not C_1, (iii) C_1 or C_2. *[NCERT]*

Solution: Given $n(C_1) = 120$, $n(C_2) = 50$, $n(C_1 \cap C_2) = 30$.
(i) C_1 but not C_2.

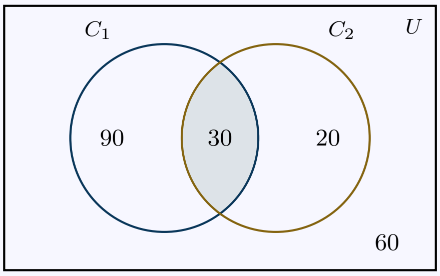
$$n(C_1 - C_2) = n(C_1) - n(C_1 \cap C_2) = 120 - 30 = 90.$$

(ii) C_2 but not C_1.

$$n(C_2 - C_1) = n(C_2) - n(C_1 \cap C_2) = 50 - 30 = 20.$$

(iii) C_1 or C_2.

$$\begin{aligned} n(C_1 \cup C_2) &= n(C_1) + n(C_2) - n(C_1 \cap C_2) \\ &= 120 + 50 - 30 \\ &= 140. \end{aligned}$$



Answer: (i) **90**, (ii) **20**, (iii) **140**.

7. **[Reasoning]** Of 500 car owners, 400 own a Maruti, 200 own a Hyundai and 50 own both. This data is

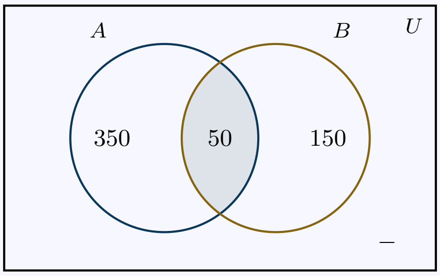
(A) consistent
(B) inconsistent, as $n(A \cup B) = 550 > 500$
(C) consistent, with 50 owning neither
(D) impossible to judge

Correct Answer: (B)

Solution: Let $A =$ Maruti, $B =$ Hyundai. Compute the union and compare with the universe of 500 owners.

$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ &= 400 + 200 - 50 \\ &= 550. \end{aligned}$$

But $n(A \cup B) \le n(U) = 500$ always must hold. Here $550 > 500$, which is impossible, so the data is *inconsistent.*



Answer: (B) — inconsistent, since $n(A \cup B) = 550 > 500$.

8. **[Minimum overlap]** In a school of 33 teachers, 15 teach Mathematics and 24 teach Physics. At least how many teach both?

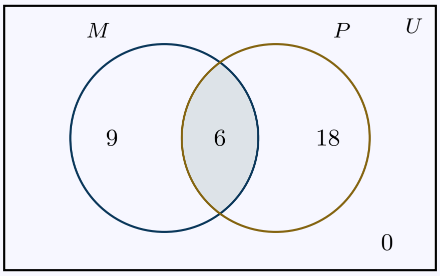
(A) 6 (B) 5 (C) 4 (D) 3

Correct Answer: (A)

Solution: The overlap is smallest when the union is largest. Since the union cannot exceed the total 33 teachers,

$$\begin{aligned} n(M \cap P) &= n(M) + n(P) - n(M \cup P) \\ &\ge 15 + 24 - 33 \\ &= 6. \end{aligned}$$

So at least 6 teachers must teach both subjects.



Answer: (A) — at least **6** teach both.

9. **[Max & min]** If $n(A) = 5$ and $n(B) = 9$ with $n(U) = 12$, the maximum and minimum values of $n(A \cup B)$ are

(A) 14, 9 (B) 12, 9 (C) 12, 5 (D) 14, 5

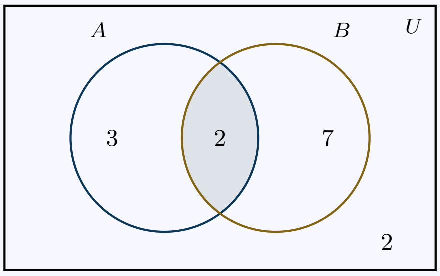
Correct Answer: (B)

Solution: In general $\max(n(A), n(B)) \le n(A \cup B) \le n(A) + n(B)$, but the union can never exceed the universe.
Maximum. The sum is $5 + 9 = 14$, but capped by $n(U) = 12$:

$$\begin{aligned} \max\, n(A \cup B) &= \min\big(n(A) + n(B),\, n(U)\big) \\ &= \min(14,\, 12) = 12. \end{aligned}$$

Minimum. The union is smallest when $A \subseteq B$, giving $n(A \cup B) = n(B)$:

$$\min\, n(A \cup B) = \max\big(n(A),\, n(B)\big) = 9.$$



Answer: (B) — maximum = 12, minimum = 9.

10. [Overlap bounds] A survey shows 63% like cheese and 76% like apples. If x% like both, then
(A) $0 \le x \le 63$
(B) $39 \le x \le 63$
(C) $39 \le x \le 76$
(D) $13 \le x \le 63$

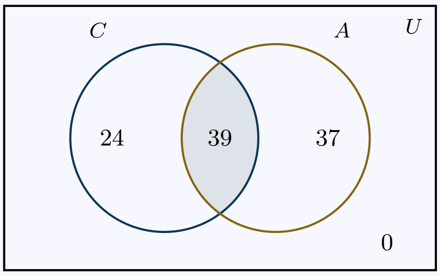
Correct Answer: (B)

Solution: Let C = cheese, A = apples (percentages), with $n(C) = 63$, $n(A) = 76$, $n(C \cap A) = x$.
Upper bound. The overlap cannot exceed either set:

$$x \le \min(63, 76) = 63.$$

Lower bound. Since the union cannot exceed 100%,

$$\begin{aligned} n(C \cup A) = 63 + 76 - x &\le 100 \\ 139 - x &\le 100 \\ x &\ge 39. \end{aligned}$$



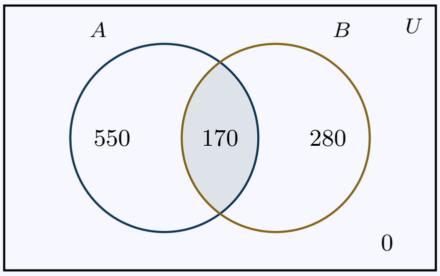
Answer: (B) — $39 \le x \le 63$.

11. [Minimum overlap] A market group surveyed 1000 consumers: 720 liked product A and 450 liked B. The least number that liked both is
(A) 170
(B) 270
(C) 450
(D) 720

Correct Answer: (A)

Solution: The overlap is least when the union is greatest, i.e. equal to the full 1000 consumers.

$$\begin{aligned} n(A\cap B) &= n(A)+n(B)-n(A\cup B) \\ &\geq 720+450-1000 \\ &= 170. \end{aligned}$$



Answer: (A) — least number liking both = **170**.

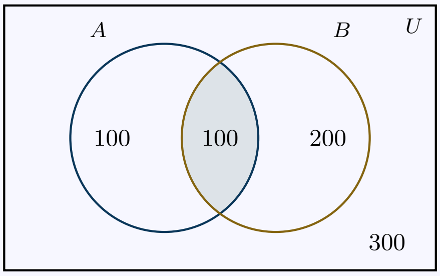
12. [**Cardinality**] If $n(U)=700$, $n(A)=200$, $n(B)=300$ and $n(A\cap B)=100$, then $n(A'\cap B')$ is

(A) 400 (B) 600 (C) 300 (D) 200

Correct Answer: (C)

Solution: By De Morgan, $A'\cap B'=(A\cup B)'$, so we count what lies outside the union.

$$\begin{aligned} n(A\cup B) &= n(A)+n(B)-n(A\cap B) \\ &= 200+300-100=400, \\ n(A'\cap B') &= n(U)-n(A\cup B) \\ &= 700-400 \\ &= 300. \end{aligned}$$



Answer: (C) — $n(A'\cap B')=300$.

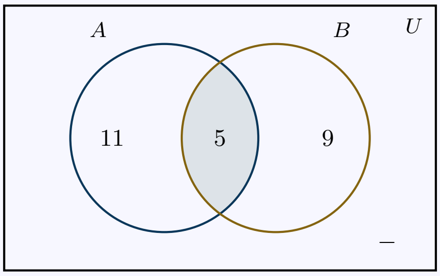
13. [**Cardinality**] If $n(A)=16$, $n(B)=14$ and $n(A\cup B)=25$, then $n(A\cap B)$ is

(A) 30 (B) 50 (C) 5 (D) 25

Correct Answer: (C)

Solution: Apply inclusion–exclusion and solve for the overlap.

$$\begin{aligned} n(A\cup B) &= n(A)+n(B)-n(A\cap B) \\ 25 &= 16+14-n(A\cap B) \\ n(A\cap B) &= 30-25 \\ n(A\cap B) &= 5. \end{aligned}$$



Answer: (C) — $n(A \cap B) = 5$.

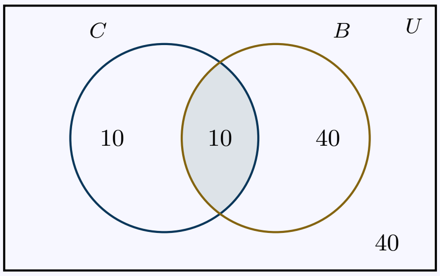
14. [Cardinality] In a city, 20% travel by car, 50% by bus and 10% by both. The percentage travelling by car or bus is

(A) 80% (B) 40% (C) 60% (D) 70%

Correct Answer: (C)

Solution: Let C = car, B = bus (percentages). "Car or bus" is the union.

$$\begin{aligned} n(C \cup B) &= n(C) + n(B) - n(C \cap B) \\ &= 20 + 50 - 10 \\ &= 60. \end{aligned}$$



Answer: (C) — **60**% travel by car or bus.

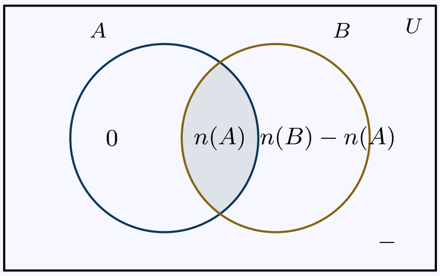
15. [Cardinality] If $A \subseteq B$, then $n(A \cap B)$ equals

(A) $n(A)$ (B) $n(B)$ (C) 0 (D) $n(A)+n(B)$

Correct Answer: (A)

Solution: If every element of A already lies in B, then $A \subseteq B$ forces the intersection to be all of A:

$$A \subseteq B \implies A \cap B = A \implies n(A \cap B) = n(A).$$



Answer: (A) — $n(A \cap B) = n(A)$.

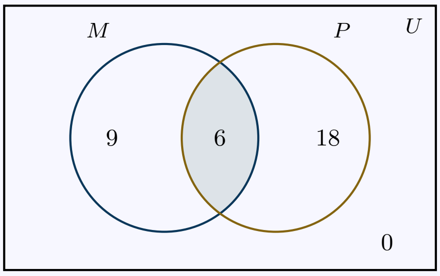
16. [**Cardinality**] Every teacher teaches Mathematics or Physics: 15 teach Maths, 24 teach Physics and 6 teach both. The number of teachers is

(A) 33 (B) 34 (C) 35 (D) 45

Correct Answer: (A)

Solution: "Maths or Physics" covers everyone, so the total equals the union.

$$\begin{aligned} n(M \cup P) &= n(M) + n(P) - n(M \cap P) \\ &= 15 + 24 - 6 \\ &= 33. \end{aligned}$$



Answer: (A) — **33** teachers.

17. [**Match the columns**] For two sets A, B, match the data in Column I with the required value in Column II.

Column I	**Column II**
(i) $n(A)=10,\ n(B)=15,\ n(A \cup B)=20$: $n(A \cap B)$	(P) 5
(ii) $n(A)=8,\ n(B)=12,\ n(A \cap B)=5$: $n(A \cup B)$	(Q) 15
(iii) $n(A \cup B)=30,\ n(A \cap B)=5,\ n(A)=18$: $n(B)$	(R) 17
(iv) $n(U)=60,\ n(A \cup B)=35$: $n((A \cup B)')$	(S) 25

Correct Answer: (i)→P, (ii)→Q, (iii)→R, (iv)→S

Solution: Apply inclusion–exclusion to each row.

$$\begin{aligned} &\text{(i)} \quad n(A \cap B) = n(A) + n(B) - n(A \cup B) = 10 + 15 - 20 = 5 && \to (P) \\ &\text{(ii)} \quad n(A \cup B) = n(A) + n(B) - n(A \cap B) = 8 + 12 - 5 = 15 && \to (Q) \\ &\text{(iii)} \quad n(B) = n(A \cup B) + n(A \cap B) - n(A) = 30 + 5 - 18 = 17 && \to (R) \\ &\text{(iv)} \quad n((A \cup B)') = n(U) - n(A \cup B) = 60 - 35 = 25 && \to (S) \end{aligned}$$

Answer: (i)→(P), (ii)→(Q), (iii)→(R), (iv)→(S).

18. [**Overlap bounds**] In a hostel, 65% of students like tea and 82% like coffee. If x% like both, then

(A) $47 \leq x \leq 65$ (B) $17 \leq x \leq 65$
(C) $47 \leq x \leq 82$ (D) $0 \leq x \leq 65$

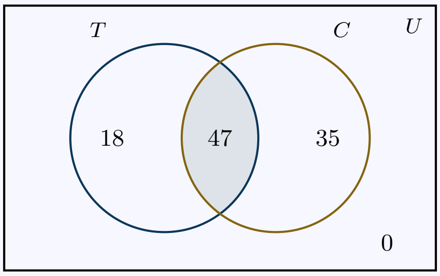
Correct Answer: (A)

Solution: Let T = tea, C = coffee (percentages), with $n(T) = 65$, $n(C) = 82$, $n(T \cap C) = x$.
Upper bound. The overlap cannot exceed the smaller set:

$$x \leq \min(65, 82) = 65.$$

Lower bound. The union cannot exceed 100%:

$$\begin{aligned} n(T \cup C) = 65 + 82 - x &\leq 100 \\ 147 - x &\leq 100 \\ x &\geq 47. \end{aligned}$$



Answer: (A) — $47 \le x \le 65$.

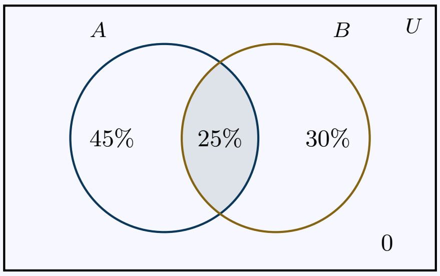
19. [**Word problem**] In a town every person reads at least one of the newspapers A and B. If 70% read A, 55% read B and 4500 people read both, find the total population of the town.

Solution: Since everyone reads at least one paper, the union is 100%. First find the "both" percentage.

$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ 100 &= 70 + 55 - n(A \cap B) \\ n(A \cap B) &= 125 - 100 \\ &= 25\%. \end{aligned}$$

So 25% of the population equals the 4500 people who read both:

$$\begin{aligned} 25\% \text{ of total} &= 4500 \\ \text{total} &= \frac{4500}{0.25} \\ &= 18000. \end{aligned}$$



Answer: Total population = **18000**.

20. [**Max & min**] If $n(A) = 25$, $n(B) = 40$ and $n(U) = 60$, the maximum and minimum possible values of $n(A \cap B)$ are

(A) 40, 5 (B) 25, 5 (C) 25, 0 (D) 40, 0

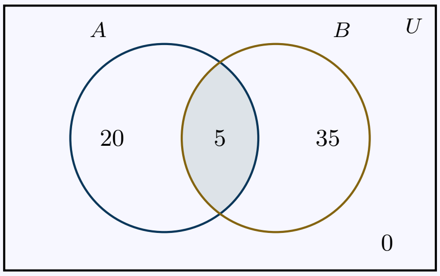
Correct Answer: (B)

Solution: The overlap is bounded by $\max(0, n(A) + n(B) - n(U)) \le n(A \cap B) \le \min(n(A), n(B))$.
Maximum. The overlap is largest when $A \subseteq B$:

$$\max\, n(A \cap B) = \min(25, 40) = 25.$$

Minimum. The overlap is smallest when the union fills the universe of 60:

$$\begin{aligned} \min\, n(A \cap B) &= \max\big(0,\ n(A) + n(B) - n(U)\big) \\ &= \max(0,\ 25 + 40 - 60) \\ &= \max(0, 5) = 5. \end{aligned}$$



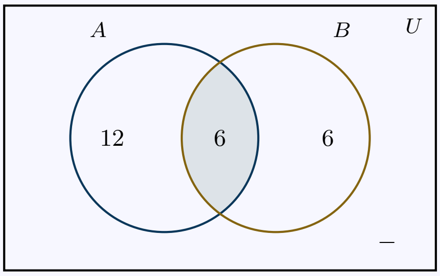
Answer: (B) — maximum $= 25$, minimum $= 5$.

21. [**Symmetric difference**] If $n(A) = 18$, $n(B) = 12$ and $n(A \cap B) = 6$, find $n(A \triangle B)$.

Solution: The symmetric difference is everything in exactly one set, so it drops the overlap twice:

$$\begin{aligned} n(A \triangle B) &= n(A) + n(B) - 2\,n(A \cap B) \\ &= 18 + 12 - 2(6) \\ &= 30 - 12 \\ &= 18. \end{aligned}$$

Equivalently, only-$A = 18 - 6 = 12$ and only-$B = 12 - 6 = 6$, summing to 18.



Answer: $n(A \triangle B) = 18$.

22. [**Word problem**] In an examination, 80% passed in English, 85% in Mathematics and 75% in both. If 40 students failed in both subjects, find the total number of students.

Solution: Let E = passed English, M = passed Maths (percentages). First find the percentage that passed at least one.

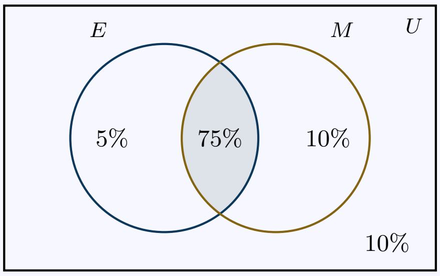
$$\begin{aligned} n(E \cup M) &= n(E) + n(M) - n(E \cap M) \\ &= 80 + 85 - 75 \\ &= 90\%. \end{aligned}$$

So the percentage failing *both* (outside the union) is

$$100\% - 90\% = 10\%.$$

This 10% equals the 40 students who failed both:

$$\begin{aligned} 10\% \text{ of total} &= 40 \\ \text{total} &= \frac{40}{0.10} \\ &= 400. \end{aligned}$$



Answer: Total number of students = **400**.