Hi everyone,

The real learning in mathematics happens when you actively engage with a problem, explore different methods, and work through challenges. Therefore, we strongly encourage you to use this solution key responsibly.

Please attempt all the problems on your own first, giving them your best and most honest effort. These solutions are to help you get unstuck on a problem after you have already tried your best.

Your effort and dedication are the true keys to success.

Sets • Assignment 05 • Solutions

Topic: Venn Diagrams & Cardinality (Three Sets)

Sub: Mathematics | **11th • JEE** | Chetan Sir

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1. **[Three-set formula]** In a class, 22 offered Mathematics, 18 Chemistry, 24 Physics; all offered at least one. If 11 offered Maths & Chemistry, 13 Chemistry & Physics, 14 Maths & Physics, and 7 all three, find (i) the total number of students, (ii) the number offering only Mathematics.

Solution: Let M, C, P be the sets of students offering Mathematics, Chemistry, Physics, with

$$n(M) = 22,\ n(C) = 18,\ n(P) = 24,\ n(M \cap C) = 11,\ n(C \cap P) = 13,\ n(M \cap P) = 14,\ n(M \cap C \cap P) = 7.$$

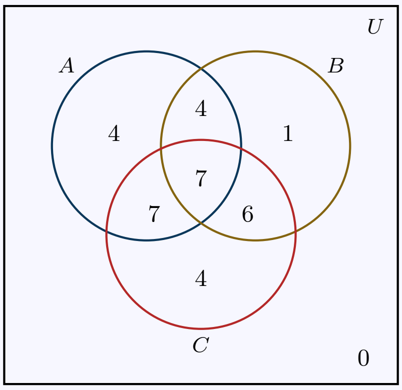
(i) By inclusion–exclusion,

$$\begin{aligned} n(M \cup C \cup P) &= n(M) + n(C) + n(P) - n(M \cap C) - n(C \cap P) - n(M \cap P) + n(M \cap C \cap P) \\ &= 22 + 18 + 24 - 11 - 13 - 14 + 7 \\ &= 64 - 38 + 7 \\ &= 33. \end{aligned}$$

(ii) Work from the centre outward. Only Mathematics:

$$\begin{aligned} n(\text{only } M) &= n(M) - n(M \cap C) - n(M \cap P) + n(M \cap C \cap P) \\ &= 22 - 11 - 14 + 7 \\ &= 4. \end{aligned}$$

Filling all eight regions (legend: A = Maths, B = Chemistry, C = Physics):



Only Maths $= 4$, only Chem $= 18 - 11 - 13 + 7 = 1$, only Phys $= 24 - 14 - 13 + 7 = 4$; Maths & Chem only $= 11 - 7 = 4$, Chem & Phys only $= 13 - 7 = 6$, Maths & Phys only $= 14 - 7 = 7$, all three $= 7$; sum $= 4 + 1 + 4 + 4 + 6 + 7 + 7 = 33$. ✓

Answer: (i) Total = **33** students. (ii) Only Mathematics = **4**.

2. **[Three-set formula]** In a survey of 60 people, 25 read newspaper H, 26 read T, 26 read I, 9 read H and I, 11 read H and T, 8 read T and I, and 3 read all three. Find (i) the number reading at least one, (ii) the number reading exactly one. *[NCERT]*

Solution: Let H, T, I be the readership sets:

$$n(H) = 25,\ n(T) = 26,\ n(I) = 26,\ n(H \cap I) = 9,\ n(H \cap T) = 11,\ n(T \cap I) = 8,\ n(H \cap T \cap I) = 3.$$

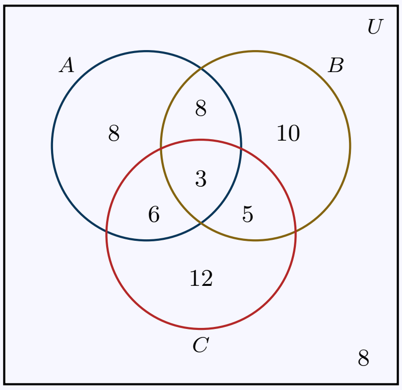
(i) Number reading at least one:

$$\begin{aligned} n(H \cup T \cup I) &= 25 + 26 + 26 - 11 - 8 - 9 + 3 \\ &= 77 - 28 + 3 \\ &= 52. \end{aligned}$$

(ii) Build the Venn from the centre. The "only" regions are

$$\begin{aligned} n(\text{only } H) &= 25 - 11 - 9 + 3 = 8, \\ n(\text{only } T) &= 26 - 11 - 8 + 3 = 10, \\ n(\text{only } I) &= 26 - 9 - 8 + 3 = 12. \end{aligned}$$

Hence exactly one $= 8 + 10 + 12 = 30$. The completed diagram (legend: $A = H$, $B = T$, $C = I$; outside $= 60 - 52 = 8$):



Pair-only: $H \cap T$ only $= 11 - 3 = 8$, $T \cap I$ only $= 8 - 3 = 5$, $H \cap I$ only $= 9 - 3 = 6$; all three $= 3$.

Answer: (i) At least one $= \mathbf{52}$. (ii) Exactly one $= \mathbf{30}$.

3. [Medals] A college awarded 38 medals in Football, 15 in Basketball and 20 in Cricket. These went to a total of 58 men, and only 3 got medals in all three sports. How many received medals in exactly two of the three sports? *[NCERT]*

Solution: Let F, B, C be the medal sets, $n(F) = 38,\ n(B) = 15,\ n(C) = 20$, with $n(F \cup B \cup C) = 58$ men and $n(F \cap B \cap C) = 3$. By inclusion–exclusion,

$$\begin{aligned} n(F \cup B \cup C) &= n(F) + n(B) + n(C) - \sum n(F \cap B) + n(F \cap B \cap C) \\ 58 &= 38 + 15 + 20 - \sum n(F \cap B) + 3 \\ 58 &= 73 - \sum n(F \cap B) + 3 \\ \sum n(F \cap B) &= 76 - 58 = 18. \end{aligned}$$

The number receiving medals in *exactly two* sports removes the all-three count from each pairwise total once for each of the three pairs:

$$\begin{aligned} n(\text{exactly two}) &= \sum n(F \cap B) - 3\,n(F \cap B \cap C) \\ &= 18 - 3(3) \\ &= 18 - 9 = 9. \end{aligned}$$

Answer: Exactly two sports $= \mathbf{9}$ men.

4. [Survey] Among three athletic teams, 21 are in basketball, 26 in hockey and 29 in football; 14 play hockey & basketball, 15 hockey & football, 12 football & basketball, and 8 play all three. Find the total number of members.

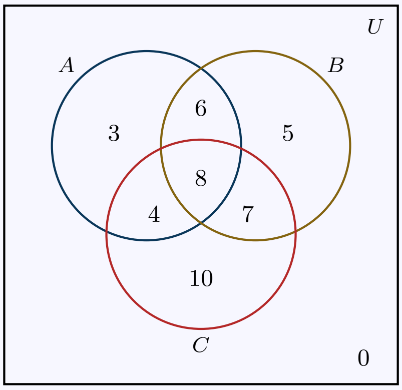
Solution: Let B, H, F be the team sets:

$$n(B) = 21,\ n(H) = 26,\ n(F) = 29,\ n(H \cap B) = 14,\ n(H \cap F) = 15,\ n(F \cap B) = 12,\ n(B \cap H \cap F) = 8.$$

By inclusion–exclusion,

$$\begin{aligned} n(B \cup H \cup F) &= n(B) + n(H) + n(F) - n(H \cap B) - n(H \cap F) - n(F \cap B) + n(B \cap H \cap F) \\ &= 21 + 26 + 29 - 14 - 15 - 12 + 8 \\ &= 76 - 41 + 8 \\ &= 43. \end{aligned}$$

Filled Venn (legend: $A = B$, $B = H$, $C = F$; outside $= 0$):



Only basketball $= 21 - 14 - 12 + 8 = 3$, only hockey $= 26 - 14 - 15 + 8 = 5$, only football $= 29 - 15 - 12 + 8 = 10$; pair-only $14 - 8 = 6$, $15 - 8 = 7$, $12 - 8 = 4$; all three $= 8$; sum $= 3 + 5 + 10 + 6 + 7 + 4 + 8 = 43$. ✓

Answer: Total members = **43**.

5. [Survey] In a survey of 100 students, the numbers studying various languages were: English only 18, English but not Hindi 23, English and Sanskrit 8, English 26, Sanskrit 48, Sanskrit and Hindi 8, no language 24. Find (i) how many studied Hindi, (ii) how many studied English and Hindi.

Solution: Let E, H, S be the language sets. The number studying at least one language is $100 - 24 = 76$. We decode the data region by region.
English only $= 18$ and "English but not Hindi" $= 23$, so the part of E outside H that is shared with S (i.e. $E \cap S$ only) is

$$n(E \cap S \text{ only}) = 23 - 18 = 5.$$

Since $n(E \cap S) = 8$, the centre is

$$n(E \cap H \cap S) = 8 - 5 = 3.$$

Now $n(E) = 26 = 18 + n(E \cap H \text{ only}) + 5 + 3$, giving $n(E \cap H \text{ only}) = 0$. Hence

$$\textbf{(ii)}\quad \begin{aligned} n(E \cap H) &= n(E \cap H \text{ only}) + n(E \cap H \cap S) \\ &= 0 + 3 = 3. \end{aligned}$$

From $n(S \cap H) = 8$ we get $n(S \cap H \text{ only}) = 8 - 3 = 5$, and from $n(S) = 48$,

$$n(S \text{ only}) = 48 - 5 - 5 - 3 = 35.$$

Summing the 76 over all regions to find Hindi only:

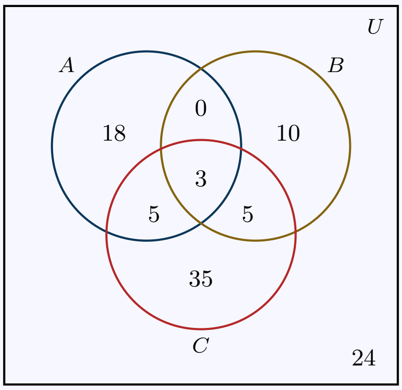
$$\begin{aligned} 76 &= \underbrace{18}_{E \text{ only}} + \underbrace{35}_{S \text{ only}} + n(H \text{ only}) + \underbrace{0}_{E \cap H} + \underbrace{5}_{H \cap S} + \underbrace{5}_{E \cap S} + \underbrace{3}_{\text{all}} \\ 76 &= 66 + n(H \text{ only}) \implies n(H \text{ only}) = 10. \end{aligned}$$

Therefore the number who studied Hindi is

$$\textbf{(i)}\quad n(H) = n(H \text{ only}) + n(E \cap H \text{ only}) + n(H \cap S \text{ only}) + n(E \cap H \cap S)$$

$$= 10 + 0 + 5 + 3 = 18.$$

Completed Venn (legend: $A = E$, $B = H$, $C = S$; outside $= 24$):



Answer: (i) Hindi $= \mathbf{18}$. (ii) English and Hindi $= \mathbf{3}$.

6. **[Minimum overlap]** In a battle, 70% of soldiers lost an eye, 85% a leg and 80% an arm. What is the minimum percentage who lost all three?

Solution: Let E, L, A be the sets (as percentages of all soldiers), with $n(E) = 70$, $n(L) = 85$, $n(A) = 80$. Since every soldier is part of at most 100%,

$$n(E \cup L \cup A) \le 100.$$

A clean lower bound for the triple overlap comes from applying the two-set bound twice. For any two sets,

$$n(X \cap Y) \ge n(X) + n(Y) - 100.$$

First combine E and L:

$$n(E \cap L) \ge 70 + 85 - 100 = 55.$$

Now combine this with A (treating $E \cap L$ as one set):

$$\begin{aligned} n(E \cap L \cap A) &\ge n(E \cap L) + n(A) - 100 \\ &\ge 55 + 80 - 100 \\ &= 35. \end{aligned}$$

Equivalently, $n(\text{all three}) \ge n(E) + n(L) + n(A) - 2(100) = 235 - 200 = 35$. This minimum is attainable.

Answer: Minimum who lost all three $= \mathbf{35}\%$.

7. **[Maximum overlap]** In a group of 40 students, 20 passed Mathematics, 25 Physics, 16 Chemistry; at most 11 passed Maths & Physics, at most 15 Physics & Chemistry, at most 15 Maths & Chemistry; each passed at least one. Find the maximum number who passed all three.

Solution: Let M, P, C be the pass-sets with $n(M) = 20$, $n(P) = 25$, $n(C) = 16$ and $n(M \cup P \cup C) = 40$ (each passed at least one). Write $t = n(M \cap P \cap C)$ and $\Sigma = \sum n(M \cap P)$ for the sum of the three pairwise intersections. By inclusion–exclusion,

$$\begin{aligned} 40 &= 20 + 25 + 16 - \Sigma + t \\ 40 &= 61 - \Sigma + t \\ \Sigma &= 21 + t. \end{aligned}$$

Each pairwise intersection contains the triple overlap, so $n(M \cap P) \ge t$, $n(P \cap C) \ge t$, $n(M \cap C) \ge t$, giving

$$\Sigma \ \ge \ 3t.$$

Substituting $\Sigma = 21 + t$:

$$21 + t \ge 3t$$

$$\begin{aligned} 21 &\geq 2t \\ t \leq 10.5 &\implies t \leq 10 \quad (t \text{ integer}). \end{aligned}$$

The bound $t = 10$ is achievable: take $n(M \cap P) = 11$, $n(P \cap C) = 10$, $n(M \cap C) = 10$ (so $\Sigma = 31 = 21 + 10$, all within the "at most" limits). Then every "only" region is non-negative — e.g. only Maths $= 20 - 11 - 10 + 10 = 9 \geq 0$, only Physics $= 25 - 11 - 10 + 10 = 14 \geq 0$, only Chemistry $= 16 - 10 - 10 + 10 = 6 \geq 0$ — so this configuration is valid.

Answer: Maximum who passed all three $=$ **10**.

8. **[Double counting]** There are 50 students; each reads 10 newspapers and each newspaper is read (shared) by exactly 20 students. Find the number of newspapers.

Solution: Count the total number of (student, newspaper) reading pairs in two ways. From the students' side, each of the 50 students reads 10 newspapers:

$$\text{total incidences} = 50 \times 10 = 500.$$

From the newspapers' side, if there are N newspapers and each is read by 20 students:

$$\text{total incidences} = N \times 20.$$

Equating the two counts,

$$\begin{aligned} 20\,N &= 500 \\ N &= \frac{500}{20} = 25. \end{aligned}$$

Answer: Number of newspapers $=$ **25**.

9. **[Survey]** A survey of 300 students found 125 like cricket, 145 football, 90 tennis; 32 like exactly two of the three games and each likes at least one. Find the number who like exactly one game.

Solution: Let C, F, T be the sets with $n(C) = 125$, $n(F) = 145$, $n(T) = 90$, and $n(C \cup F \cup T) = 300$ (each likes at least one). Let e_1, e_2, e_3 be the numbers liking exactly one, exactly two, exactly three games; we are given $e_2 = 32$.
Two relations hold. The total of the singles $n(C) + n(F) + n(T)$ counts each "exactly two" student twice and each "exactly three" student three times:

$$\begin{aligned} n(C) + n(F) + n(T) &= e_1 + 2e_2 + 3e_3 \\ 125 + 145 + 90 &= e_1 + 2(32) + 3e_3 \\ 360 &= e_1 + 64 + 3e_3. \qquad (1) \end{aligned}$$

Also, the union counts each student once:

$$\begin{aligned} e_1 + e_2 + e_3 &= 300 \\ e_1 + 32 + e_3 &= 300 \\ e_1 + e_3 &= 268. \qquad (2) \end{aligned}$$

From (1): $e_1 + 3e_3 = 296$. Subtracting (2):

$$\begin{aligned} (e_1 + 3e_3) - (e_1 + e_3) &= 296 - 268 \\ 2e_3 &= 28 \implies e_3 = 14. \end{aligned}$$

Then from (2), $e_1 = 268 - 14 = 254$.

Answer: Exactly one game $=$ **254**.

10. [MCQ] In a class of 50, 22 like Maths, 18 Physics, 20 Chemistry; each likes at least one and no one likes all three. The number liking exactly two subjects is

(A) 8 (B) 10 (C) 12 (D) 15

Correct Answer: (B)

Solution: Let M, P, C with $n(M) = 22,\ n(P) = 18,\ n(C) = 20$, $n(M \cup P \cup C) = 50$ and $n(M \cap P \cap C) = 0$ (no one likes all three). By inclusion–exclusion,

$$\begin{aligned} n(M \cup P \cup C) &= n(M) + n(P) + n(C) - \sum n(M \cap P) + n(M \cap P \cap C) \\ 50 &= 22 + 18 + 20 - \sum n(M \cap P) + 0 \\ 50 &= 60 - \sum n(M \cap P) \\ \sum n(M \cap P) &= 10. \end{aligned}$$

With no triple overlap, every student in a pairwise intersection lies in *exactly* two subjects, so

$$\begin{aligned} n(\text{exactly two}) &= \sum n(M \cap P) - 3\,n(M \cap P \cap C) \\ &= 10 - 3(0) = 10. \end{aligned}$$

Answer: (B) 10.

11. [MCQ] For three sets, $n(A \cup B \cup C)$ equals

(A) $n(A) + n(B) + n(C)$
(B) $\sum n(A) - \sum n(A \cap B) + n(A \cap B \cap C)$
(C) $\sum n(A) + n(A \cap B \cap C)$
(D) None of these

Correct Answer: (B)

Solution: The inclusion–exclusion principle for three sets is

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C),$$

which in compact notation is exactly

$$n(A \cup B \cup C) = \sum n(A) - \sum n(A \cap B) + n(A \cap B \cap C).$$

Option (A) over-counts the overlaps, and (C) omits the subtraction of the pairwise intersections.

Answer: (B).

12. [MCQ] The number of elements belonging to exactly two of A, B, C is

(A) $\sum n(A \cap B) - 3\,n(A \cap B \cap C)$
(B) $\sum n(A \cap B) - 2\,n(A \cap B \cap C)$
(C) $\sum n(A \cap B) - n(A \cap B \cap C)$
(D) $\sum n(A \cap B)$

Correct Answer: (A)

Solution: The sum of the three pairwise intersections $\sum n(A \cap B) = n(A \cap B) + n(B \cap C) + n(C \cap A)$ counts each "exactly two" element once, but it counts each element of the centre $A \cap B \cap C$ in all three pairs, i.e. three times. To keep only the genuinely "exactly two" elements we must remove the centre three times:

$$n(\text{exactly two}) = \sum n(A \cap B) - 3\,n(A \cap B \cap C).$$

Answer: (A).

13. [MCQ] Let $A_1, \ldots, A_{30}$ be 30 sets with 5 elements each and $B_1, \ldots, B_n$ be n sets with 3 elements each, with $\bigcup A_i = \bigcup B_j = S$. If each element of S lies in exactly 10 of the A_i and exactly 9 of the B_j, then n is

(A) 15 (B) 45 (C) 3 (D) 35

Correct Answer: (B)

Solution: Count incidences (element, set) two ways. For the A_i: there are 30 sets of 5 elements, so $30 \times 5 = 150$ incidences; and each of the $|S|$ elements lies in 10 of them:

$$\begin{aligned} 30 \times 5 &= |S| \times 10 \\ 150 &= 10\,|S| \implies |S| = 15. \end{aligned}$$

For the B_j: there are n sets of 3 elements, and each element lies in 9 of them:

$$\begin{aligned} n \times 3 &= |S| \times 9 \\ 3n &= 15 \times 9 = 135 \\ n &= 45. \end{aligned}$$

Answer: (B) 45.

14. [MCQ] In a survey of 100 students: 40 like tea, 30 coffee, 20 milk; 15 tea & coffee, 10 coffee & milk, 12 tea & milk, 5 all three. The number who like none is

(A) 42 (B) 58 (C) 37 (D) 45

Correct Answer: (A)

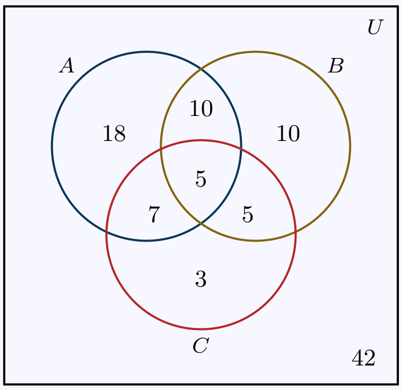
Solution: Let T, C, M be the sets, $n(T) = 40$, $n(C) = 30$, $n(M) = 20$, $n(T \cap C) = 15$, $n(C \cap M) = 10$, $n(T \cap M) = 12$, $n(T \cap C \cap M) = 5$. The number liking at least one drink is

$$\begin{aligned} n(T \cup C \cup M) &= 40 + 30 + 20 - 15 - 10 - 12 + 5 \\ &= 90 - 37 + 5 \\ &= 58. \end{aligned}$$

Out of 100 students, those who like none are

$$100 - n(T \cup C \cup M) = 100 - 58 = 42.$$

Filled Venn (legend: $A = T$, $B = C$, $C = M$; outside $= 42$):



Only tea $= 40 - 15 - 12 + 5 = 18$, only coffee $= 30 - 15 - 10 + 5 = 10$, only milk $= 20 - 10 - 12 + 5 = 3$; pair-only $10, 5, 7$; all three $= 5$.

Answer: (A) 42.

15. **[Match the columns]** Given $n(A)$=20, $n(B)$=15, $n(C)$=12, $n(A \cap B)$=5, $n(B \cap C)$=4, $n(C \cap A)$=3, $n(A \cap B \cap C)$=2, match each entry of Column I with its value in Column II.

(i) $n(A \cup B \cup C)$	(P) 2
(ii) only A	(Q) 14
(iii) exactly two of them	(R) 6
(iv) all three	(S) 37

Correct Answer: (i)→(S), (ii)→(Q), (iii)→(R), (iv)→(P)

Solution: Evaluate each quantity directly.
(i) Union:

$$\begin{aligned} n(A \cup B \cup C) &= 20 + 15 + 12 - 5 - 4 - 3 + 2 \\ &= 47 - 12 + 2 = 37 \quad \to \ (S). \end{aligned}$$

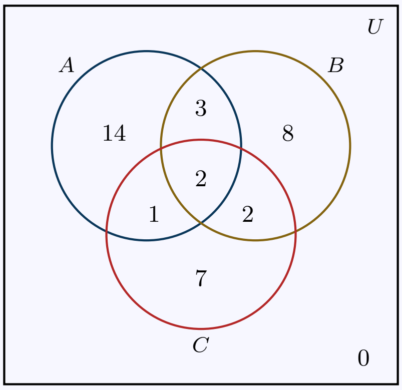
(ii) Only A:

$$\begin{aligned} n(\text{only } A) &= n(A) - n(A \cap B) - n(C \cap A) + n(A \cap B \cap C) \\ &= 20 - 5 - 3 + 2 = 14 \quad \to \ (Q). \end{aligned}$$

(iii) Exactly two:

$$\begin{aligned} n(\text{exactly two}) &= \sum n(A \cap B) - 3\,n(A \cap B \cap C) \\ &= (5 + 4 + 3) - 3(2) = 12 - 6 = 6 \quad \to \ (R). \end{aligned}$$

(iv) All three $= n(A \cap B \cap C) = 2 \ \to \ (P)$.
The completed Venn (only $A = 14$, only $B = 15 - 5 - 4 + 2 = 8$, only $C = 12 - 4 - 3 + 2 = 7$; pair-only $5 - 2 = 3,\ 4 - 2 = 2,\ 3 - 2 = 1$; centre 2; outside 0):



Answer: (i)→(S), (ii)→(Q), (iii)→(R), (iv)→(P).

16. **[Minimum overlap]** In a survey of 100 students, 60 like Mathematics, 70 Physics and 80 Chemistry. Find the minimum number of students who like all three subjects.

Solution: Let M, P, C with $n(M) = 60,\ n(P) = 70,\ n(C) = 80$, out of a universe of 100. Apply the two-set lower bound $n(X \cap Y) \geq n(X) + n(Y) - 100$ twice.
First combine P and C:

$$n(P \cap C) \geq 70 + 80 - 100 = 50.$$

Now combine with M:

$$\begin{aligned} n(M \cap P \cap C) &\geq n(M) + n(P \cap C) - 100 \\ &\geq 60 + 50 - 100 = 10. \end{aligned}$$

Equivalently $n(\text{all three}) \geq n(M) + n(P) + n(C) - 2(100) = 210 - 200 = 10$, and this value is attainable.

Answer: Minimum who like all three = **10**.

17. [Survey] A survey of 500 TV viewers found: 285 watch Football, 195 Hockey, 115 Basketball, 45 Football & Basketball, 70 Football & Hockey, 50 Hockey & Basketball, and 50 watch none. Find (i) how many watch all three games, (ii) how many watch exactly one game.

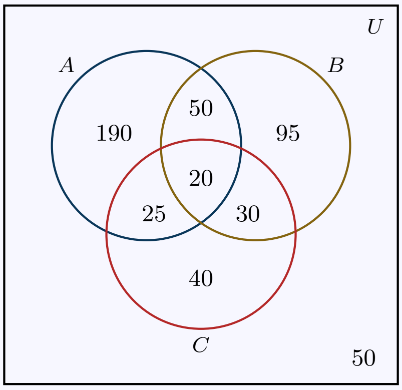
Solution: Let F, H, B be the sets, $n(F) = 285$, $n(H) = 195$, $n(B) = 115$, $n(F \cap B) = 45$, $n(F \cap H) = 70$, $n(H \cap B) = 50$. Since 50 watch none, the number watching at least one is $500 - 50 = 450$.
(i) Let $x = n(F \cap H \cap B)$. By inclusion–exclusion,

$$\begin{aligned} n(F \cup H \cup B) &= 285 + 195 + 115 - 70 - 50 - 45 + x \\ 450 &= 595 - 165 + x \\ 450 &= 430 + x \implies x = 20. \end{aligned}$$

(ii) The "only" regions (working outward from the centre $x = 20$):

$$\begin{aligned} n(\text{only } F) &= 285 - 70 - 45 + 20 = 190, \\ n(\text{only } H) &= 195 - 70 - 50 + 20 = 95, \\ n(\text{only } B) &= 115 - 45 - 50 + 20 = 40. \end{aligned}$$

Exactly one $= 190 + 95 + 40 = 325$. Completed Venn (legend: $A = F$, $B = H$, $C = B$; outside $= 50$):



Pair-only: $F \cap H$ only $= 70 - 20 = 50$, $H \cap B$ only $= 50 - 20 = 30$, $F \cap B$ only $= 45 - 20 = 25$; all three $= 20$.

Answer: (i) All three $=$ **20**. (ii) Exactly one $=$ **325**.

18. [Survey] In a class of 200 students, 120 passed Mathematics, 90 Physics, 70 Chemistry, 40 Maths & Physics, 30 Physics & Chemistry, 35 Maths & Chemistry and 15 passed all three. Find (i) the number who passed exactly one subject, (ii) the number who passed at least two subjects.

Solution: Let M, P, C with $n(M) = 120$, $n(P) = 90$, $n(C) = 70$, $n(M \cap P) = 40$, $n(P \cap C) = 30$, $n(M \cap C) = 35$, $n(M \cap P \cap C) = 15$.
(i) The "only" regions, working outward from the centre:

$$\begin{aligned} n(\text{only } M) &= 120 - 40 - 35 + 15 = 60, \\ n(\text{only } P) &= 90 - 40 - 30 + 15 = 35, \\ n(\text{only } C) &= 70 - 35 - 30 + 15 = 20. \end{aligned}$$

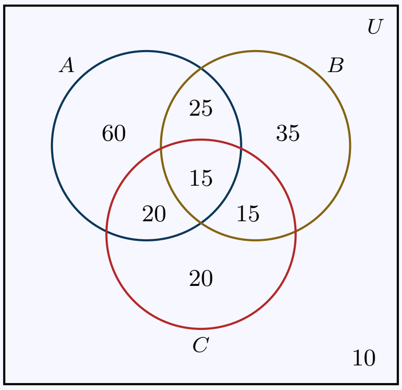
Exactly one $= 60 + 35 + 20 = 115$.
(ii) First the union:

$$\begin{aligned} n(M \cup P \cup C) &= 120 + 90 + 70 - 40 - 30 - 35 + 15 \\ &= 280 - 105 + 15 = 190. \end{aligned}$$

"At least two" $=$ (at least one) $-$ (exactly one):

$$n(\text{at least two}) = 190 - 115 = 75.$$

Completed Venn (legend: $A = M$, $B = P$, $C = C$; outside $= 200 - 190 = 10$):



Pair-only: $M \cap P$ only $= 40 - 15 = 25$, $P \cap C$ only $= 30 - 15 = 15$, $M \cap C$ only $= 35 - 15 = 20$; all three $= 15$. Check at-least-two $= 25 + 15 + 20 + 15 = 75$. ✓

Answer: (i) Exactly one $=$ **115**. (ii) At least two $=$ **75**.

19. [Three-set formula] If $n(A \cup B \cup C) = 100$, $n(A) = n(B) = n(C) = 50$ and $n(A \cap B) = n(B \cap C) = n(C \cap A) = 20$, find $n(A \cap B \cap C)$.

Solution: Substitute into inclusion–exclusion and solve for the centre. Let $x = n(A \cap B \cap C)$:

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - \sum n(A \cap B) + n(A \cap B \cap C)$$
$$100 = (50 + 50 + 50) - (20 + 20 + 20) + x$$
$$100 = 150 - 60 + x$$
$$100 = 90 + x \implies x = 10.$$

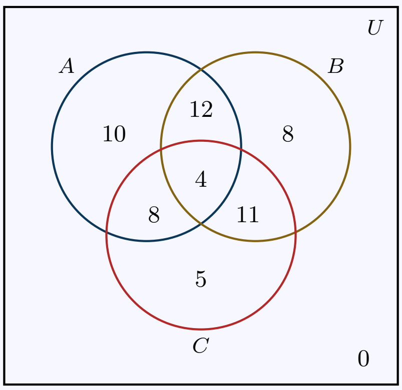
Answer: $n(A \cap B \cap C) =$ **10**.

20. [Exactly two] For three sets, $n(A \cup B \cup C) = 58$; the numbers in "only A", "only B" and "only C" are 10, 8 and 5, and $n(A \cap B \cap C) = 4$. Find the number of elements belonging to exactly two of the sets.

Solution: Partition the union into its three disjoint pieces: exactly one, exactly two, and exactly three. Let e_2 denote the number in exactly two sets. Then

$$n(A \cup B \cup C) = (\text{exactly one}) + (\text{exactly two}) + (\text{exactly three})$$
$$58 = \underbrace{(10 + 8 + 5)}_{=23} + e_2 + \underbrace{4}_{\text{all three}}$$
$$58 = 27 + e_2 \implies e_2 = 31.$$

A representative completed Venn (only-regions $10, 8, 5$; centre 4; the three pair-only regions summing to 31, e.g. $12, 11, 8$; outside 0):



Answer: Exactly two of the sets $=$ **31**.