Hi everyone,

The real learning in mathematics happens when you actively engage with a problem, explore different methods, and work through challenges. Therefore, we strongly encourage you to use this solution key responsibly.

Please attempt all the problems on your own first, giving them your best and most honest effort. These solutions are to help you get unstuck on a problem after you have already tried your best.

Your effort and dedication are the true keys to success.

Sets • Assignment 06 • Solutions

Topic: JEE Main PYQ (Sets & Venn Diagrams)

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Section A : Sets Defined by a Property

1. **[Integer Type]** The number of elements in the set $\{n \in \mathbb{Z} : |n^2 - 10n + 19| < 6\}$ is ______. *[JEE Main 2023]*

Solution: Remove the modulus: $|n^2 - 10n + 19| < 6 \Leftrightarrow -6 < n^2 - 10n + 19 < 6$.

$$\textbf{Lower:}\quad n^2 - 10n + 19 > -6 \;\Rightarrow\; n^2 - 10n + 25 > 0 \;\Rightarrow\; (n-5)^2 > 0 \;\Rightarrow\; n \neq 5.$$
$$\textbf{Upper:}\quad n^2 - 10n + 19 < 6 \;\Rightarrow\; n^2 - 10n + 13 < 0 \;\Rightarrow\; n \in \left(5 - 2\sqrt{3},\; 5 + 2\sqrt{3}\right).$$

Now $5 - 2\sqrt{3} \approx 1.54$ and $5 + 2\sqrt{3} \approx 8.46$, so the integers satisfying the upper bound are $2, 3, 4, 5, 6, 7, 8$. Discarding $n = 5$:

$$\{2, 3, 4, 6, 7, 8\}.$$

Answer: 6 elements.

2. **[Integer Type]** Let $X = \{n \in \mathbb{N} : 1 \leq n \leq 50\}$, let A be the multiples of 2 in X and B the multiples of 7 in X. The number of elements in the smallest subset of X containing both A and B is ______. *[JEE Main 2020]*

Solution: The smallest subset containing both A and B is $A \cup B$.

$$n(A) = \left\lfloor \tfrac{50}{2} \right\rfloor = 25, \qquad n(B) = \left\lfloor \tfrac{50}{7} \right\rfloor = 7,$$
$$n(A \cap B) = \#\{\text{multiples of } 14 \leq 50\} = 3 \qquad (14, 28, 42).$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B) = 25 + 7 - 3 = 29.$$

Answer: 29.

3. **[Integer Type]** The number of natural numbers n with $10 \leq n \leq 100$ for which $3^n - 3$ is a multiple of 7 is ______. *[JEE Main 2023]*

Solution: Work with $3^n \pmod 7$. The powers cycle with period 6:

$$3^1 \equiv 3,\; 3^2 \equiv 2,\; 3^3 \equiv 6,\; 3^4 \equiv 4,\; 3^5 \equiv 5,\; 3^6 \equiv 1 \pmod 7.$$

We need $3^n \equiv 3 \pmod 7$, i.e. $n \equiv 1 \pmod 6$.

$$n \in \{13, 19, 25, \ldots, 97\} \;\cap\; [10, 100].$$

This is an AP with first term 13, common difference 6, last term 97:

$$\text{count} = \frac{97 - 13}{6} + 1 = 14 + 1 = 15.$$

Answer: 15.

4. [MCQ] In a class of 140 students numbered 1 to 140, all even-numbered students opted Mathematics, those whose number is divisible by 3 opted Physics, and those divisible by 5 opted Chemistry. The number of students who did not opt for any of the three courses is

(A) 42 (B) 102 (C) 1 (D) 38

Correct Answer: (D) *[JEE Main 2019]*

Solution: "None of the three" means not divisible by 2, 3 or 5. By inclusion–exclusion on $\{1,\ldots,140\}$:

$$\begin{aligned}|2\cup 3\cup 5| &= \left(\tfrac{140}{2}+\tfrac{140}{3}+\tfrac{140}{5}\right)-\left(\tfrac{140}{6}+\tfrac{140}{10}+\tfrac{140}{15}\right)+\tfrac{140}{30}\\ &=(70+46+28)-(23+14+9)+4\\ &=144-46+4=102.\end{aligned}$$

Hence those opting nothing $=140-102=38$.

Answer: (D) 38.

5. [MCQ] Let A be the set of natural numbers from 100 to 700 (inclusive) that are neither multiples of 3 nor multiples of 4. The number of elements in A is

(A) 300 (B) 310 (C) 290 (D) 280

Correct Answer: (A) *[JEE Main 2024]*

Solution: Total integers from 100 to 700 inclusive $=700-100+1=601$. Count multiples of 3, of 4, and of 12 in $[100,700]$:

$$\begin{aligned}\text{mult of }3:\ \left\lfloor\tfrac{700}{3}\right\rfloor-\left\lfloor\tfrac{99}{3}\right\rfloor &= 233-33=200,\\ \text{mult of }4:\ \left\lfloor\tfrac{700}{4}\right\rfloor-\left\lfloor\tfrac{99}{4}\right\rfloor &= 175-24=151,\\ \text{mult of }12:\ \left\lfloor\tfrac{700}{12}\right\rfloor-\left\lfloor\tfrac{99}{12}\right\rfloor &= 58-8=50.\end{aligned}$$

$$\text{mult of 3 or 4}=200+151-50=301\ \Rightarrow\ n(A)=601-301=300.$$

Answer: (A) 300.

6. [Integer Type] Let $A=\{n\in\mathbb{N}\mid n^2\le n+10000\}$, $B=\{3k+1\mid k\in\mathbb{N}\}$ and $C=\{2k\mid k\in\mathbb{N}\}$. The sum of all the elements of $A\cap(B-C)$ is ______. *[JEE Main 2021]*

Solution: First pin down each set.

$$\begin{aligned}A:\ n^2\le n+10000 &\Rightarrow n^2-n-10000\le 0\Rightarrow n\le \tfrac{1+\sqrt{40001}}{2}\approx 100.5,\\ &\text{so } A=\{1,2,\ldots,100\}.\end{aligned}$$

$B=\{4,7,10,13,\ldots\}$ and $C=$ even naturals. Then $B-C$ keeps only the *odd* members of B:

$$3k+1\text{ is odd}\Leftrightarrow k\text{ even}\Rightarrow B-C=\{7,13,19,25,\ldots\}=\{6m+1:m\in\mathbb{N}\}.$$

Intersect with A: the elements $7,13,\ldots,97$ (i.e. $6m+1\le 100\Rightarrow m=1,\ldots,16$). Summing,

$$\sum_{m=1}^{16}(6m+1)=6\cdot\frac{16\cdot 17}{2}+16=6(136)+16=816+16=832.$$

Answer: 832.

7. [MCQ] Let $A = \{1, 2, 3, \ldots, 10\}$ and let B be the set of all fractions $\frac{m}{n}$ with $m, n \in A$, $m < n$ and $\gcd(m, n) = 1$. Then $n(B)$ equals

(A) 29 (B) 31 (C) 37 (D) 36

Correct Answer: (B) *[JEE Main 2025]*

Solution: Because each fraction is in lowest terms ($\gcd = 1$) with $m < n$, distinct pairs give distinct values, so $n(B)$ is just the number of valid pairs. For a fixed denominator n, the number of $m < n$ with $\gcd(m, n) = 1$ is Euler's $\varphi(n)$:

$$\begin{aligned} n(B) &= \sum_{n=2}^{10} \varphi(n) \\ &= \varphi(2) + \varphi(3) + \cdots + \varphi(10) \\ &= 1 + 2 + 2 + 4 + 2 + 6 + 4 + 6 + 4 = 31. \end{aligned}$$

Answer: (B) 31.

Section B : Subsets, Power Set & Counting

8. [Integer Type] Set A has m elements and set B has n elements. If the total number of subsets of A is 112 more than that of B, then $m \cdot n$ is ______. *[JEE Main 2020]*

Solution: A set with k elements has 2^k subsets, so

$$2^m - 2^n = 112$$
$$2^n\left(2^{m-n} - 1\right) = 112 = 2^4 \cdot 7 = 16 \cdot 7.$$

Matching the odd factor, $2^{m-n} - 1 = 7 \Rightarrow 2^{m-n} = 8 \Rightarrow m - n = 3$, and $2^n = 16 \Rightarrow n = 4$, so $m = 7$.

$$m \cdot n = 7 \times 4 = 28.$$

Answer: 28.

9. [MCQ] Let A and B be finite sets with m and n elements. The total number of subsets of A is 56 more than that of B. Then the distance of $P(m, n)$ from $Q(-2, -3)$ is

(A) 8 (B) 10 (C) 4 (D) 6

Correct Answer: (B) *[JEE Main 2024]*

Solution:

$$2^m - 2^n = 56 = 2^3 \cdot 7 = 64 - 8 = 2^6 - 2^3$$
$$\Rightarrow \; m = 6, \; n = 3 \Rightarrow P = (6, 3).$$

Distance from $Q(-2, -3)$:

$$PQ = \sqrt{(6 - (-2))^2 + (3 - (-3))^2} = \sqrt{8^2 + 6^2} = \sqrt{100} = 10.$$

Answer: (B) 10.

10. [Integer Type] Let $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{3, 6, 7, 9\}$. The number of sets C with $C \subseteq A$ and $C \cap B \neq \varnothing$ is ______. *[JEE Main 2022]*

Solution: Use the complement. The total number of subsets of A is $2^7 = 128$. Now $C \cap B = \varnothing$ means C avoids every element of B that lies in A, namely $\{3, 6, 7\}$ (note $9 \notin A$). So such C are subsets of $\{1, 2, 4, 5\}$:

$$\#\{C : C \cap B = \varnothing\} = 2^4 = 16,$$
$$\#\{C : C \cap B \neq \varnothing\} = 128 - 16 = 112.$$

Answer: 112.

11. [MCQ] Let $X = \{1, 2, 3, 4, 5\}$. The number of ordered pairs (Y, Z) with $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z = \varnothing$ is

(A) 3^5 (B) 2^5 (C) 5^3 (D) 5^2

Correct Answer: (A) *[JEE Main 2012]*

Solution: Decide, *independently for each element* of X, where it goes. Since $Y \cap Z = \varnothing$, no element may be in both, so each of the 5 elements has exactly 3 choices:

$$\text{in } Y \text{ only}, \quad \text{in } Z \text{ only}, \quad \text{in neither}.$$

By the multiplication principle the number of ordered pairs is $3 \times 3 \times 3 \times 3 \times 3 = 3^5$.

Answer: (A) 3^5 $(= 243)$.

12. [Integer Type] Let $A = \{1, 2, 3, 4, 5, 6, 7\}$. Let B be the collection of subsets T of A for which $1 \notin T$ or $2 \in T$, and C the collection of subsets T for which the sum of the elements of T is a prime number. The number of elements in $B \cup C$ is ______. *[JEE Main 2022]*

Solution: Use $n(B \cup C) = n(B) + n(C) - n(B \cap C)$ on the $2^7 = 128$ subsets of A.
Count $n(B)$. The statement "$1 \notin T$ or $2 \in T$" fails only when $1 \in T$ *and* $2 \notin T$. Fixing 1 in and 2 out, the other 5 elements are free:

$$n(B) = 128 - 2^5 = 128 - 32 = 96.$$

Count $n(C)$. We count subsets whose element–sum is one of the primes $\leq 1 + 2 + \cdots + 7 = 28$. Grouping by the prime value of the sum:

sum	2	3	5	7	11	13	17	19	23
#subsets	1	2	3	5	7	8	7	6	3

$$n(C) = 1 + 2 + 3 + 5 + 7 + 8 + 7 + 6 + 3 = 42.$$

Count $n(B \cap C)$ (prime sum *and* in B): a direct check of the prime-sum subsets removes the 11 that have $1 \in T, 2 \notin T$, leaving $n(B \cap C) = 42 - 11 = 31$. Therefore

$$n(B \cup C) = 96 + 42 - 31 = 107.$$

(Equivalently: a subset is outside $B \cup C$ iff $1 \in T$, $2 \notin T$ and the sum is non-prime; there are exactly 21 such subsets, and $128 - 21 = 107$.)

Answer: 107.

13. [Integer Type] For $n \geq 2$, let S_n be the set of all subsets of $\{1, 2, \ldots, n\}$ containing no two consecutive numbers. Then $n(S_5)$ equals ______. *[JEE Main 2025]*

Solution: Let a_n be the number of subsets of $\{1, \ldots, n\}$ with no two consecutive elements (the empty set counts). Conditioning on whether n is used:

$$a_n = \underbrace{a_{n-1}}_{n \text{ not used}} + \underbrace{a_{n-2}}_{n \text{ used, so } n-1 \text{ excluded}},$$

with $a_0 = 1,\ a_1 = 2$. This is the Fibonacci recurrence:

$$a_0 = 1,\ a_1 = 2,\ a_2 = 3,\ a_3 = 5,\ a_4 = 8,\ a_5 = 13.$$

Answer: $n(S_5) = 13$.

14. [MCQ] Let $S = \{1, 2, \ldots, 100\}$. The number of non-empty subsets A of S for which the product of the elements of A is even is

(A) $2^{50} - 1$
(B) $2^{50}\left(2^{50} - 1\right)$
(C) $2^{100} - 1$
(D) $2^{50} + 1$

Correct Answer: (B) *[JEE Main 2019]*

Solution: The product is *odd* exactly when every chosen element is odd. S has 50 odd and 50 even numbers.

$$\begin{aligned} \#\{\text{product odd}\} &= 2^{50} \quad (\text{any subset of the 50 odd numbers, incl. } \varnothing), \\ \#\{\text{product even}\} &= 2^{100} - 2^{50}. \end{aligned}$$

The empty set sits in the "odd" group (empty product $= 1$), so no further adjustment is needed:

$$2^{100} - 2^{50} = 2^{50}\left(2^{50} - 1\right).$$

Answer: (B) $2^{50}\left(2^{50} - 1\right)$.

15. [Integer Type] Let S be the set of the first 11 natural numbers. The number of subsets B of S with $n(B) \geq 2$ for which the product of all elements of B is even is ______. *[JEE Main 2026]*

Solution: $S = \{1, \ldots, 11\}$ has 6 odd and 5 even numbers. First count *all* even-product subsets (at least one even element):

$$\#\{\text{product even}\} = 2^{11} - 2^{6} = 2048 - 64 = 1984.$$

We need $n(B) \geq 2$, so subtract even-product subsets of size 0 or 1:

$$\begin{aligned} &\text{size } 0 : \text{the empty set has odd product} \Rightarrow 0 \text{ removed}, \\ &\text{size } 1 : \text{even-product singletons are the 5 even numbers} \Rightarrow 5 \text{ removed}. \end{aligned}$$

$$1984 - 5 = 1979.$$

Answer: 1979.

Section C : Operations & Algebra of Sets

16. [MCQ] Let $\bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^{n} Y_i = T$, where each X_i has 10 elements and each Y_i has 5 elements. If each element of T belongs to exactly 20 of the X_i and exactly 6 of the Y_i, then n equals

(A) 30
(B) 50
(C) 15
(D) 45

Correct Answer: (A) *[JEE Main 2020]*

Solution: Count the pairs (element, set it lies in) two ways.

$$\text{From the } X_i : \quad 50 \times 10 = 20 \cdot n(T) \;\Rightarrow\; n(T) = \frac{500}{20} = 25.$$

$$\text{From the } Y_i : \quad n \times 5 = 6 \cdot n(T) = 6 \times 25 = 150 \;\Rightarrow\; n = \frac{150}{5} = 30.$$

Answer: (A) 30.

17. [MCQ] If A, B, C are three sets with $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then

(A) $A = C$ (B) $B = C$ (C) $A \cap B = \varnothing$ (D) $A = B$

Correct Answer: (B) *[JEE Main 2009]*

Solution: Start from B and use the two hypotheses:

$$\begin{aligned} B &= B \cap (A \cup B) = B \cap (A \cup C) && (\text{since } A \cup B = A \cup C) \\ &= (B \cap A) \cup (B \cap C) = (A \cap C) \cup (B \cap C) && (\text{since } A \cap B = A \cap C) \\ &= (A \cup B) \cap C = (A \cup C) \cap C = C. \end{aligned}$$

Answer: (B) $B = C$.

18. [MCQ] Let A, B, C be sets such that $\varnothing \neq A \cap B \subseteq C$. Then which statement is **not** true?

(A) If $(A - B) \subseteq C$, then $A \subseteq C$ (B) $B \cap C \neq \varnothing$

(C) $(C \cup A) \cap (C \cup B) = C$ (D) If $(A - C) \subseteq B$, then $A \subseteq B$

Correct Answer: (D) *[JEE Main 2019]*

Solution: We are given $A \cap B \subseteq C$ and $A \cap B \neq \varnothing$. Test each option.

(A) $A = (A - B) \cup (A \cap B)$; if $A - B \subseteq C$ and $A \cap B \subseteq C$ then $A \subseteq C$. **True.**

(B) $A \cap B \neq \varnothing$ and $A \cap B \subseteq C$, so some element lies in both B and C. **True.**

(C) $(C \cup A) \cap (C \cup B) = C \cup (A \cap B) = C$ since $A \cap B \subseteq C$. **True.**

(D) $A = (A - C) \cup (A \cap C)$. Even if $A - C \subseteq B$, the part $A \cap C$ need not lie in B, so $A \subseteq B$ can fail. **Not necessarily true.**

Answer: (D) is not true.

19. [MCQ] Let $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$. Then

(A) $P \subset Q$ and $Q - P = \varnothing$ (B) $Q \subset P$

(C) $P \subset Q$ (D) $P = Q$

Correct Answer: (D) *[JEE Main 2016]*

Solution: Reduce each defining condition to a single trigonometric equation.

$$P: \ \sin\theta - \cos\theta = \sqrt{2}\cos\theta \Rightarrow \sin\theta = (\sqrt{2}+1)\cos\theta \Rightarrow \tan\theta = \sqrt{2}+1.$$

$$Q: \ \sin\theta + \cos\theta = \sqrt{2}\sin\theta \Rightarrow \cos\theta = (\sqrt{2}-1)\sin\theta \Rightarrow \tan\theta = \frac{1}{\sqrt{2}-1} = \sqrt{2}+1.$$

Both sets are exactly $\{\theta : \tan\theta = \sqrt{2}+1\}$, so $P = Q$.

Answer: (D) $P = Q$.

20. [MCQ] Let A be the set of real m for which both roots of $x^2 - (m+1)x + m + 4 = 0$ are real, and $B = [-3, 5)$. Which is **not** true?

(A) $A \cap B = \{-3\}$ (B) $B - A = (-3, 5)$

(C) $A \cup B = \mathbb{R}$ (D) $A - B = (-\infty, -3) \cup (5, \infty)$

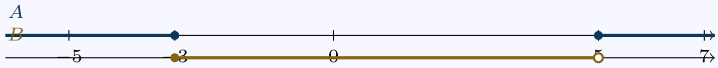
Correct Answer: (D) *[JEE Main 2020]*

Solution: Both roots are real $\Leftrightarrow$ discriminant ≥ 0:

$$\begin{aligned} (m+1)^2 - 4(m+4) &\geq 0 \\ m^2 - 2m - 15 &\geq 0 \end{aligned}$$

$$(m-5)(m+3) \geq 0 \;\Rightarrow\; m \leq -3 \text{ or } m \geq 5.$$

So $A = (-\infty, -3] \cup [5, \infty)$ and $B = [-3, 5)$.



Now $A - B$ removes from A everything in B. Only -3 of A lies in B, while $5 \in A$ but $5 \notin B$, so 5 *stays*:

$$A - B = (-\infty, -3) \cup [5, \infty) \;\neq\; (-\infty, -3) \cup (5, \infty).$$

Options (A) $A \cap B = \{-3\}$, (B) $B - A = (-3, 5)$, (C) $A \cup B = \mathbb{R}$ are all correct; (D) wrongly drops the point 5.

Answer: (D) is not true.

Section D : Venn Diagrams & Surveys

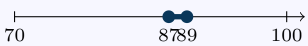
21. **[MCQ]** Out of all patients in a hospital, 89% suffer from heart ailment and 98% from lungs infection. If K% suffer from both, then K **cannot** belong to

(A) $\{80, 83, 86, 89\}$
(B) $\{84, 86, 88, 90\}$
(C) $\{79, 81, 83, 85\}$
(D) $\{84, 87, 90, 93\}$

Correct Answer: (C) *[JEE Main 2021]*

Solution: With $n(H) = 89$, $n(L) = 98$ (as percentages of ≤ 100):

$$\max\big(0,\, 89 + 98 - 100\big) \leq K \leq \min(89, 98)$$
$$87 \leq K \leq 89.$$



A valid K must lie in $[87, 89]$. Sets (A), (B), (D) each contain a number in this band (89, 88, 87), but every element of (C) $\{79, 81, 83, 85\}$ is below 87.

Answer: (C) — K cannot lie in $\{79, 81, 83, 85\}$.

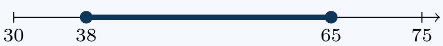
22. **[MCQ]** A survey shows 73% of an office likes coffee and 65% likes tea. If x% like both, then x **cannot** be

(A) 63 (B) 36 (C) 54 (D) 38

Correct Answer: (B) *[JEE Main 2020]*

Solution:

$$\max\big(0,\, 73 + 65 - 100\big) \leq x \leq \min(73, 65)$$
$$38 \leq x \leq 65.$$



$63, 54, 38$ all lie in $[38, 65]$, but $36 < 38$.

Answer: (B) — x cannot be 36.

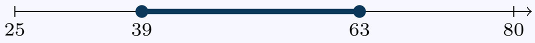
23. **[MCQ]** A survey shows 63% read newspaper A and 76% read B. If x% read both, a possible value of x is

(A) 37 (B) 65 (C) 29 (D) 55

Correct Answer: (D) *[JEE Main 2020]*

Solution:

$$\max\big(0,\ 63+76-100\big) \le x \le \min(63,76)$$
$$39 \le x \le 63.$$



Among the options only 55 lies in $[39, 63]$ ($37, 29$ are too small, 65 too large).

Answer: (D) $x = 55$ is possible.

24. [MCQ] An organization awarded 48 medals in event A, 25 in B and 18 in C. If these went to a total of 60 men with only 5 getting medals in all three events, the number receiving medals in exactly two events is

(A) 10 (B) 15 (C) 21 (D) 9

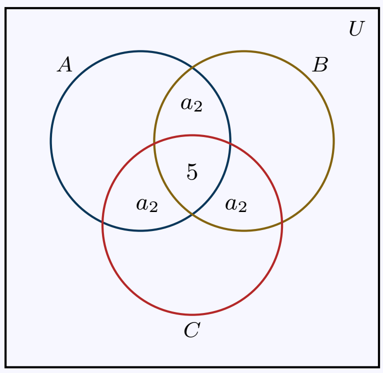
Correct Answer: (C) *[JEE Main 2023]*

Solution: Let a_1, a_2, a_3 be the numbers of men with exactly one, two and three medals. Then "60 men" and "$48 + 25 + 18 = 91$ medals" give

$$a_1 + a_2 + a_3 = 60,$$
$$a_1 + 2a_2 + 3a_3 = 91, \qquad a_3 = 5.$$

Substituting $a_3 = 5$: $a_1 + a_2 = 55$ and $a_1 + 2a_2 = 76$. Subtracting,

$$a_2 = 76 - 55 = 21.$$



(a_1 = exactly one event, the three lens regions sum to $a_2 = 21$, centre = 5)

Answer: (C) 21.

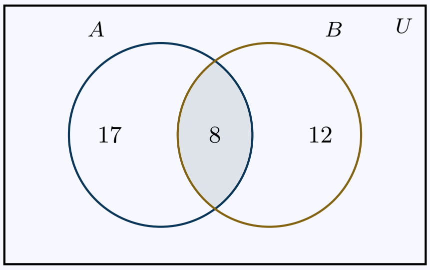
25. [MCQ] Newspapers A, B are published in a city: 25% read A, 20% read B, 8% read both. Of those who read A but not B, 30% look at ads; of B but not A, 40%; of both, 50%. The % of the population looking at ads is

(A) 13.5 (B) 13 (C) 12.8 (D) 13.9

Correct Answer: (D) *[JEE Main 2019]*

Solution: Split the readers into disjoint regions:

$$\text{only } A = 25 - 8 = 17, \qquad \text{only } B = 20 - 8 = 12, \qquad \text{both} = 8.$$



Now apply the ad-viewing rates to each region:

$$\begin{aligned}\text{ads} &= 0.30(17) + 0.40(12) + 0.50(8)\\ &= 5.1 + 4.8 + 4.0 = 13.9\%.\end{aligned}$$

Answer: (D) 13.9%.

26. [**Integer Type**] In a survey of 220 students, at least 125 and at most 130 studied Mathematics; at least 85 and at most 95 Physics; at least 75 and at most 90 Chemistry; 30 studied Physics & Chemistry; 50 Chemistry & Mathematics; 40 Mathematics & Physics; and 10 studied none. If m, n are the least and most who studied all three, then $m + n$ is ______. *[JEE Main 2024]*

Solution: Since 10 studied none, $n(M \cup P \cup C) = 220 - 10 = 210$. Let $t = n(M \cap P \cap C)$. By inclusion–exclusion (the pairwise counts $40, 30, 50$ each include t),

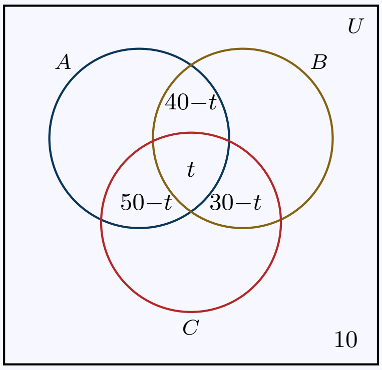
$$\begin{aligned}210 &= M + P + C - (40 + 30 + 50) + t\\ \Rightarrow \quad t &= 330 - (M + P + C).\end{aligned}$$

Upper bound on t. Each "exactly two" region is non-negative:

$$40 - t \geq 0, \quad 30 - t \geq 0, \quad 50 - t \geq 0 \;\Rightarrow\; t \leq 30.$$

Range of $M+P+C$. With $M \leq 130, P \leq 95, C \leq 90$ we get $M+P+C \leq 315$, so $t = 330-(M+P+C) \geq 15$. The value $t \leq 30$ forces $M + P + C \geq 300$, which is attainable (e.g. $M = 130, P = 95, C = 75$). Hence

$$t_{\min} = 15 \text{ (at } M{+}P{+}C{=}315), \qquad t_{\max} = 30 \text{ (at } M{+}P{+}C{=}300).$$



So $m = 15$, $n = 30$ and $m + n = 45$.

Answer: $m + n = 45$.