

ANSWERS

1. DIFFERENTIATION

EXERCISE 1.1

- (1) (i) $5(3x^2 - 2)(x^3 - 2x - 1)^4$
 (ii) $\frac{5}{2}(3\sqrt{x} - 4\sqrt[3]{x})(2x^{\frac{3}{2}} - 3x^{\frac{4}{3}} - 5)^{\frac{3}{2}}$
 (iii) $\frac{x+2}{\sqrt{x^2+4x-7}}$
 (iv) $\frac{x(2\sqrt{x^2+1}+1)}{2\sqrt{x^2+1} \cdot \sqrt{x^2+\sqrt{x^2+1}}}$
 (v) $-\frac{4x-7}{(2x^2-7x-5)^{\frac{8}{3}}}$
 (vi) $\frac{15(3x-4)}{2(3x-5)^{\frac{3}{2}}} \left(\sqrt{3x-5} - \frac{1}{\sqrt{3x-5}} \right)^4$
- (2) (i) $-2x \sin(x^2 + a^2)$ (ii) $\frac{3e^{(3x+2)}}{2\sqrt{e^{(3x+2)}+5}}$
 (iii) $\operatorname{cosec} x$ (iv) $\frac{\sec^2 \sqrt{x}}{4\sqrt{x} \cdot \sqrt{\tan \sqrt{x}}}$
 (v) $\frac{-9 \operatorname{cosec}^2[\log(x^3)] \cdot \cot^2[\log(x^3)]}{x}$
 (vi) $3 \sin^2 x \cdot \cos x \cdot 5^{\sin^3 x + 3} \cdot \log 5$
 (vii) $\frac{\sin x \operatorname{cosec} \sqrt{\cos x} \cdot \cot(\sqrt{\cos x})}{2\sqrt{\cos x}}$
 (viii) $-3x^2 \tan(x^3 - 5)$
 (ix) $5 \sin 2x \cdot e^{3 \sin^2 x - 2 \cos^2 x}$
 (x) $\frac{-2x \cdot \sin[2 \log(x^2 + 7)]}{x^2 + 7}$
 (xi) $-\sec^2[\cos(\sin x)] \cdot \sin(\sin x) \cdot \cos x$
 (xii) $4x^3 \cdot \sec^2(x^4 + 4) \cdot \sec[\tan(x^4 + 4)] \cdot \tan[\tan(x^4 + 4)]$

- (xiii) $\frac{2 \log x}{x} - \frac{2}{x}$
 (xiv) $\frac{\cos \sqrt{\sin \sqrt{x}} \cdot \cos \sqrt{x}}{4\sqrt{x} \cdot \sqrt{\sin \sqrt{x}}}$
 (xv) $2x \cdot e^{x^2} [\tan(e^{x^2})]$ (xvi) $\frac{1}{2x \log x}$
 (xvii) $\frac{2[\log[\log(\log x)]]}{x \log x \cdot \log(\log x)}$
 (xviii) $4x \sin(2x^2)$
- (3) (i) $6(x+2)(x^2+4x+1)^2 + 4(3x^2-5)(x^3-5x-2)^3$
 (ii) $8(1-2x)(1+4x)^5(3+x-x^2)^7 + 20(1+4x)^4(3+x-x^2)^8$
 (iii) $\frac{14-3x}{2(7-3x)^{\frac{3}{2}}}$ (iv) $\frac{6x^2(x^3+15)(x^3-5)^4}{(x^3+3)^6}$
 (v) $\sin 2x(1+\sin^2 x)(1+\cos^2 x)^2(1-5\sin^2 x)$
 (vi) $-\frac{\sin x}{2\sqrt{\cos x}} - \frac{\sin \sqrt{x}}{4\sqrt{x} \cdot \sqrt{\cos \sqrt{x}}}$
 (vii) $3 \sec 3x$ (viii) $\frac{\pi \cos x^\circ}{90(1-\sin x^\circ)^2}$
 (ix) $-\frac{\operatorname{cosec}^2\left(\frac{\log x}{2}\right)}{2x} + \tan x \cdot \operatorname{cosec}^2 x$
 (x) $\frac{8e^{4x}}{(e^{4x}+1)^2}$ (xi) $-\frac{e^{\sqrt{x}}}{\sqrt{x}(e^{\sqrt{x}}-1)^2}$
 (xii) $6 \operatorname{cosec} 2x + 4 \cot x + \frac{14x}{x^2+7}$
 (xiii) $3 \operatorname{cosec} 3x$ (xiv) $-\frac{5}{2} \operatorname{cosec}\left(\frac{5x}{2}\right)$

(xv) $-\sec x$

(xvi) $2 \log 4 + \frac{3x}{x^2 + 5} - \frac{9x^2}{2(2x^3 - 4)}$

(xvii) $2x - \frac{6}{5 - 4x} + \frac{2}{7 - 6x}$

(xviii) $-\sin x \log a - \frac{6x}{x^2 - 3} - \frac{1}{x \log x}$

(xix) 0 (xx) $\frac{x(x^2 + 2)^3(7x^2 + 38)}{(x^2 + 5)^{\frac{3}{2}}}$

(4) (i) -16 (ii) 35 (iii) -20 (iv) 28

(5) -5 (6) $\frac{12}{5}$ (7) $x = 0$ or $\frac{2\pi}{3}$ or 2π

(8) $e^{2x} + 6e^x + 14, e^{x^2+5}, 2x, e^x, f'[g(x)] \cdot g'(x), 2e^{2x} + 6e^x, 8, g'[f(x)] \cdot f'(x), 2xe^{x^2+5}, -2e^6.$

EXERCISE 1.2

(1) (i) $\frac{1}{2\sqrt{x}}$ (ii) $-\frac{1}{4\sqrt{x}\sqrt{2-\sqrt{x}}}$

(iii) $\frac{1}{3\sqrt[3]{(x-2)^2}}, \text{ for } x > 2$ (iv) $\frac{2}{2x-1}$

(v) 2 (vi) e^x

(vii) $2e^{2x-3}$ (viii) $\frac{1}{x \log 2}$

(2) (i) $\frac{1}{x \cdot e^x(x+2)}$ (ii) $\frac{1}{\cos x - x \sin x}$

(iii) $\frac{1}{7^x(x \log 7 + 1)}$ (iv) $\frac{x}{2x^2 + 1}$

(v) $\frac{1}{1 + \log x}$

(3) (i) $\frac{1}{14}$ (ii) $\frac{1}{4}$ (iii) $\frac{1}{12}$ (iv) $\frac{1}{5}$

(4) $\frac{1}{4}$

(5) (i) and (ii) derivative proved.

(6) (i) $\frac{1}{x[1 + (\log x)^2]}$ (ii) $\frac{e^x}{\sqrt{1 - e^{2x}}}$

(iii) $-\frac{3x^2}{1 + x^6}$ (iv) $-\frac{4^x \log 4}{1 + 4^{2x}}$

(v) $\frac{1}{2\sqrt{x}(1+x)}$ (vi) $\frac{x}{\sqrt{1-x^4}}$

(vii) $\frac{2}{\sqrt{2-x^2}}$ (viii) $\frac{3\sqrt{x}}{2\sqrt{1-x^3}}$

(ix) $9x^8$ (x) $2x$

(7) (i) $2xe^{x^2}$ (ii) $-5^x \log 5$ (iii) $\frac{1}{2}$

(iv) $-x$ (v) $-\frac{1}{2}$ (vi) -6

(vii) $-\frac{1}{6}$ (viii) $-\frac{3}{2}$ (ix) $-\frac{7}{2}$

(x) $-\frac{1}{2}$ (xi) $-\frac{1}{2}$ (xii) $\frac{2}{3}$

(8) (i) 1 (ii) 1 (iii) $\frac{1}{2\sqrt{x}}$

(iv) 3 (v) e^x (vi) $2^x \log 2$

(9) (i) $\frac{2}{1+x^2}$ (ii) $\frac{2}{1+x^2}$

(iii) $-\frac{2}{1+x^2}$ (iv) $\pm \frac{2}{\sqrt{1-x^2}}$

(v) $-\frac{3}{\sqrt{1-x^2}}$ (vi) $-\frac{2e^x}{1+e^{2x}}$

(vii) $\frac{2 \cdot 3^x \log 3}{1 + 3^{2x}}$

(viii) $\frac{2 \cdot 4^x \log 4}{1 + 4^{2x}}$ or $\left(\frac{4^{x+\frac{1}{2}} \log 4}{1 + 4^{2x}} \right)$

(ix) $-\frac{10}{1 + 25x^2}$ (x) $-\frac{3\sqrt{x}}{1 + x^3}$

(xi) $\frac{5x\sqrt{x}}{1 + x^5}$ (xii) $\frac{1}{2\sqrt{x}(1+x)}$

(10) (i) $\frac{3}{1 + 9x^2} + \frac{5}{1 + 25x^2}$

$$(ii) \frac{7}{1+49x^2} - \frac{5}{1+25x^2}$$

$$(iii) \frac{1}{2\sqrt{x}} \left(\frac{3}{1+9x} - \frac{1}{1+x} \right)$$

$$(iv) 2^x \log 2 \left(\frac{3}{1+9(2^{2x})} + \frac{1}{1+2^{2x}} \right)$$

$$(v) 2^x \log 2 \left(\frac{2}{1+4(2^{2x})} - \frac{1}{1+2^{2x}} \right)$$

$$(vi) \frac{3a}{a^2+9x^2} + \frac{2a}{a^2+4x^2} \quad (vii) 1$$

$$(viii) \frac{2}{1+(2x+1)^2} - \frac{3}{1+(3x-4)^2}$$

$$(ix) \frac{2}{1+(2x+3)^2} + \frac{1}{1+(x-1)^2}$$

EXERCISE 1.3

$$(1) (i) \frac{(x+1)^2}{(x+3)^3(x+3)^4} \left[\frac{2}{x+1} - \frac{3}{x+2} - \frac{4}{x+3} \right]$$

$$(ii) \frac{1}{3} \sqrt[3]{\frac{4x-1}{(2x+3)(5-2x)^2}} \left(\frac{4}{4x-1} - \frac{2}{2x+3} + \frac{4}{5-2x} \right)$$

$$(iii) (x^2+3)^{\frac{3}{2}} \cdot \sin^3 2x \cdot 2^{x^2} \left[\frac{3x}{x^2+3} + 6 \cot 2x + 2x \log 2 \right]$$

$$(iv) \frac{(x^2+2x+2)^{\frac{3}{2}}}{(\sqrt{x}+3)^3(\cos x)^x} \left[\frac{3(x+1)}{x^2+2x+2} - \frac{3}{2\sqrt{x}(\sqrt{x}+3)} + x \tan x - \log(\cos x) \right]$$

$$(v) \frac{x^5 \cdot \tan^3 4x}{\sin^2 3x} \left[\frac{5}{x} + 24 \operatorname{cosec} 8x - 6 \cot 3x \right]$$

$$(vi) x^{\tan^{-1} x} \left[\frac{\tan^{-1} x}{x} + \frac{\log x}{1+x^2} \right]$$

$$(vii) \sin^x x \quad [x \cot x + \log(\sin x)]$$

$$(viii) \cos(x^x) \cdot x^x(1 + \log x)$$

$$(2) (i) ex^{e^{-1}} + e^x + x^x(1 + \log x) \quad (ii) x^{x^x} \cdot x^x \cdot \log x \left[1 + \log x + \frac{1}{x \log x} \right] + e^{x^x} \cdot x^x(1 + \log x)$$

$$(iii) (\log x)^x \left[\frac{1}{\log x} + \log(\log x) \right] + (\cos x)^{\cot x} [1 + \operatorname{cosec}^2 x \log(\cos x)]$$

$$(iv) x^{e^x} \cdot e^x \left[\frac{1}{x} + \log x \right] + (\log x)^{\sin x} \left[\frac{\sin x}{x \log x} + \cos x \log(\log x) \right]$$

$$(v) \sec^2 x \cdot e^{\tan x} + (\log x)^{\tan x} \left[\frac{\tan x}{x \log x} + \sec^2 x \log(\log x) \right]$$

$$(vi) (\sin x)^{\tan x} [1 + \sec^2 x \log(\sin x)] + (\cos x)^{\cot x} [1 + \operatorname{cosec}^2 x \log(\cos x)]$$

$$(vii) 10^{x^x} x^x \log 10 (1 + \log x) + x^{x^{10}} \cdot x^9 (1 + 10 \log x) + x^{10^x} \cdot 10^x \left(\frac{1}{x} + \log x \cdot \log 10 \right)$$

$$(viii) 2$$

(3) (i) $-\sqrt{\frac{y}{x}}$ (ii) $-\sqrt{\frac{x}{y}}$
 (iii) $-\frac{\sqrt{y}(2\sqrt{x}+\sqrt{y})}{\sqrt{x}(2\sqrt{y}+\sqrt{x})}$ (iv) $-\frac{3x^2+2xy+y^2}{x^2+2xy+3y^2}$
 (v) $-\frac{y}{x}$ (vi) $-\frac{e^y+ye^x}{e^x+xe^y}$
 (vii) $\frac{\sin(x-y)+e^{x+y}}{\sin(x-y)-e^{x+y}}$ (viii) $-\frac{1+y\sin(xy)}{1+x\sin(xy)}$
 (ix) $\frac{y(1-xe^{x-y})}{x(1-ye^{x-y})}$
 (x) $\frac{\sin(x-y)-\cos(x+y)-1}{\sin(x-y)+\cos(x+y)-1}$

EXERCISE 1.4

(1) (i) $\frac{1}{t}$ (ii) $\frac{b}{a}\cos\theta$ (iii) $\frac{2}{\sqrt{a^2+m^2}}$
 (iv) $\sec^3\theta$ (v) $\frac{b}{a}\tan\left(\frac{\theta}{2}\right)$
 (vi) $\frac{y(t^2+1)\log a}{axt}$ (vii) $-\frac{1}{2}$ (viii) $\frac{1}{3}$
 (2) (i) $\frac{3\sqrt{3}}{2}$ (ii) $-\sqrt{3}$ (iii) $-\frac{\pi}{6}$
 (iv) $1-\sqrt{2}$ (v) $3+\pi$
 (4) (i) $\frac{x\cos x+\sin x}{\sec^2 x}$ (ii) 1
 (iii) $-\frac{1}{2}$ (iv) 2 (v) $-x(\log x)^2 \cdot 3^x$
 (vi) $-\frac{x\sqrt{x^2-1}}{2}$
 (vii) $\frac{(1+\log x) \cdot x^{x+1-\sin x}}{\sin x+x\cos x \cdot \log x}$
 (viii) $\frac{\sqrt{1-x^2}}{4(1+x^2)}$

EXERCISE 1.5

(1) (i) $40x^3-24x-\frac{12}{x^4}$
 (ii) $2e^{2x}(1+\tan x) \cdot (2+\tan x+\tan^2 x)$
 (iii) $-e^{4x}(9\cos 5x+40\sin 5x)$
 (iv) $x(5+6\log x)$ (v) $-\frac{1+\log x}{(x\log x)^2}$
 (iv) $x^{x-1}+x^x(1+\log x)^2$
 (2) (i) $-\frac{1}{4a}\operatorname{cosec}^4\left(\frac{\theta}{2}\right)$ (ii) $-\frac{1}{4at^3}$
 (iii) 6 (iv) $-\frac{2\sqrt{2}b}{a^2}$
 (4) (i) $\frac{d^ny}{dx^n} = \frac{m!a^n(ax+b)^{m-n}}{(m-n)!}$ if $m>0, m>n$,
 $\frac{d^ny}{dx^n} = 0$ if $m>0, m<n$
 $\frac{d^ny}{dx^n} = n!a^n$ if $m>0, m=n$
 (ii) $\frac{(-1)^n n!}{x^{n+1}}$ (iii) a^ne^{ax+b}
 (iv) $p^na^{px+q}(\log a)^n$
 (v) $\frac{(-1)^{n-1}(n-1)!a^n}{(ax+b)^n}$ (vi) $\cos\left(\frac{n\pi}{2}+x\right)$
 (vii) $a^n\sin\left(\frac{n\pi}{2}+ax+b\right)$
 (viii) $(-2)^n\cos\left(\frac{n\pi}{2}+3-2x\right)$
 (ix) $\frac{(-1)^{n-1}(n-1)!2^n}{(2x+3)^n}$
 (x) $\frac{(-1)^n \cdot n! \cdot 3^n}{(3x-5)^{n+1}}$
 (xi) $e^{ax}(a^2+b^2)^{\frac{n}{2}} \cdot \cos\left[bx+c+n\tan^{-1}\left(\frac{b}{a}\right)\right]$
 (xii) $e^{8x} \cdot (10)^n \cos\left[6x+7+n\tan^{-1}\left(\frac{3}{4}\right)\right]$

MISCELLANEOUS EXERCISE 1

(I)

1	2	3	4	5	6	7	8	9	10	11	12
D	C	C	B	A	C	D	C	B	C	A	B

(II) (1) $\frac{3}{4}$ (ii) Does not exist (iii) -2

(2) (A) 3, (B) 5, (C) 4, (D) 1.

(3) (i) $-\frac{1}{9}$ (ii) $-\frac{40}{3}$ (iii) $-\frac{29}{96}$

(iv) $-\frac{4}{9}$

(4) (i) $-\frac{x}{\sqrt{1-x^2}}$ [Hint : $x = \cos 2\theta$]

(ii) $-\frac{1}{2}$ [Hint : $x = \cos 2\theta$]

(iii) $\frac{3}{2\sqrt{x}(1+x)}$ [Hint : $\sqrt{x} = \tan \theta$]

(iv) $-\frac{1}{2\sqrt{1-x^2}}$ [Hint : $x = \cos 2\theta$]

(v) $\frac{3}{1+9x^2} + \frac{5}{1+25x^2}$

(vi) $\frac{1}{2(1+x^2)}$ [Hint : $x = \tan \theta$]

(6) (i) $\frac{\sqrt{1-x^2}}{4(1+x^2)}$

(ii) $-\frac{2x}{\sqrt{1+x^2} \cdot \sin(\log x)}$ (iii) 1

2. APPLICATIONS OF DERIVATIVES

EXERCISE 2.1

(1) (i) $2x - y + 4 = 0, x + 2y - 8 = 0$

(ii) $4x - 5y + 12 = 0, 5x + 4y - 26 = 0,$

(iii) $y = 2, x = \sqrt{3}$

(iv) $\pi x + 2y - 2\pi = 0,$

$4x - 2\pi y + \pi^2 - 4 = 0$

(v) $2x - y = 0, 4x + 8y - 5\pi = 0$

(vi) $4x + 2y - 3 = 0, 2x - 4y + 1 = 0$

(vii) $17x - 4y - 20 = 0, 8x + 34y - 135 = 0$

(2) (4, 1)

(3) $(2, -2) \left(-\frac{2}{3}, -\frac{14}{27} \right)$ (4) $y = 0$ and $y = 4$

(5) $x + 3y - 8 = 0, x + 3y + 8 = 0$

(6) $a = 2, b = -7$ (7) $(4, 11)$ and $\left(-4, -\frac{31}{3} \right)$

(8) $0.8 \pi \text{ cm}^2/\text{sec}.$ (9) $6 \text{ cm}^3/\text{sec}.$

(10) $\frac{3\sqrt{6}}{2} \text{ cm}^2/\text{sec}.$ (11) $8 \text{ cm}^2/\text{sec}$

(12) $7.2 \text{ cm}^3/\text{sec}$ (13) 3 km/hr

(14) (i) $\left(\frac{3}{8} \right) \text{ meter/sec}.$ (ii) $\frac{9}{8} \text{ meter/sec}.$

(15) $0.9 \text{ meter/sec}.$ (16) $\left(\frac{4\pi}{3} \right) \text{ cm}^3/\text{sec}$

EXERCISE 2.2

- (1) (i) 2.9168 (ii) 3.03704 (iii) 1.9997
(iv) 248.32 (v) 64.48
- (2) (i) 0.953 (ii) 0.42423 (iii) 0.4924
(iv) 1.02334
- (3) (i) 0.7845 (ii) 0.7859 (iii) 0.7859
- (4) (i) 2.70471 (ii) 8.1279 (iii) 9.09887
- (5) (i) 4.6152 (ii) 2.1983 (iii) 3.006049
- (6) (i) 6.91 (ii) 9.72

EXERCISE 2.3

- (1) (i) Valid (ii) Valid
(iii) Invalid (iv) Valid
(v) Invalid (vi) Invalid
- (2) $b = 1$
- (3) (i) $\frac{\pi}{4}$ or $\frac{5\pi}{4}$ (ii) $c = \pi$ (iii) $c = \frac{5}{2}$
- (4) $p = -6, q = 11$ (6) $c = -2$
- (7) (i) $e - 1$ (ii) $2 \pm \frac{2}{\sqrt{3}}$ (iii) $\frac{1}{7}$
(iv) $\frac{1}{2}$ (v) $3 + \sqrt{2}$

EXERCISE 2.4

- (1) (i) Increasing $\forall x \in R$
(ii) Decreasing $\forall x \in R$
(iii) Increasing $\forall x \in R$
- (2) (i) $x < -1$ and $x > 2$ (ii) $R - \{1\}$
(iii) $x < -2$ and $x > 6$
- (3) (i) $-1 < x < 2$ (ii) $(-5, 5) - \{0\}$
(iii) $x \in (2, 4)$
- (4) (a) $(-\infty, -4] \cup [12, \infty)$
(b) $-4 \leq x \leq 12$ i.e. $[-4, 12]$
- (5) (a) $x < -3$ and $x > 8$ (b) $-3 < x < 8$
- (6) (a) $-1 < x < 1$ (b) $(-\infty, -1) \cup (1, \infty)$
- (9) (i) $\text{Max} = \frac{36}{25}, \text{Min} = -\frac{16}{27}$
(ii) $\text{Max} = -3, \text{Min} = -128$
(iii) $\text{Max} = 20, \text{Min} = 16$ (iv) $\text{Min} = 8$
(v) $\text{Min} = -\frac{1}{e}$ (vi) $\text{Max} = \frac{1}{e}$
- (10) 15, 15 (11) 10, 10 (12) 9 (13) 12.8
- (14) $l = \sqrt{2}$ and $b = \frac{1}{\sqrt{2}}$
- (15) Radius = Height = a (16) 3, 3
- (17) Side of square base = 8 cm, Height = 4 cm
- (18) $x = 75, P = 4000$ (19) 6, 9
- (22) $\frac{4\pi r^3}{3\sqrt{3}} \text{ cm}^3$

MISCELLANEOUS EXERCISE 2

(I)

1	2	3	4	5	6	7	8	9	10
A	C	B	B	D	C	D	A	D	D

(II) (2) 4

(3) $14x - 13y + 12 = 0$, $13x + 14y - 41 = 0$

(4) $\frac{2}{9\pi}$ ft/sec (5) $\left(\frac{16}{3}, 3\right), \left(-\frac{16}{3}, -3\right)$

(6) $c = 0$ (7) $c = 2$ (8) 2.025

(9) 1.03565

(10) Decreasing in $\left(0, \frac{1}{e}\right]$ and

Increasing in $\left[\frac{1}{e}, \infty\right)$

(11) Increasing in $[e, \infty)$, Decreasing in $(1, e]$

(15) $l = \frac{60}{\pi + 4}$, $b = \frac{30}{\pi + 4}$, $r = \frac{30}{\pi + 4}$

(17) Side = $\frac{l}{\pi + 4}$, Radius = $\frac{l}{2(\pi + 4)} = \frac{x}{2}$

(18) 24, 45 (21) Max = $\frac{5}{4}$, Min = 1

3. INDEFINITE INTEGRATION

EXERCISE 3.1

(1) (i) $\frac{x^4}{4} + \frac{x^3}{3} - \frac{x^2}{2} + x + c$ (ii) $\frac{x^3}{3} - 2x^2 + 4x + c$

(iii) $3 \tan x - 4 \log x - \frac{2}{\sqrt{x}} - 7x + c$

(iv) $\frac{x^2}{4} - \frac{5x^2}{2} + 3 \log x - \frac{1}{x^4} + c$

(v) $\frac{6}{5}x^2\sqrt{x} - 4\sqrt{x} - \frac{10}{\sqrt{x}} + c$

(2) (i) $\tan x - x + c$ (ii) $-2 \cos x + c$

(iii) $\sec x + c$ (iv) $-\cot x - 2x + c$

(v) $-\cot x - \tan x + x + c$

(vi) $\sec x - \tan x + x + c$

(vii) $\sec x - \tan x + x + c$

(viii) $\sin x - \cos x + c$ (ix) $-\sqrt{2} \cos x + c$

(x) $-\frac{1}{14} \cos 7x - \frac{1}{2} \cos x + c$

(3) (i) $x - 2 \log(x + 2) + c$

(ii) $2x + \frac{1}{2} \log(2x + 1) + c$

(iii) $\frac{5}{3}x - \frac{26}{9} \log(3x - 4) + c$

(iv) $\frac{2(x+5)^{\frac{3}{2}}}{3} - 14\sqrt{x+5} + c$

(v) $\frac{1}{12}(4x-1)^{\frac{3}{2}} - \frac{13}{4}\sqrt{4x-1} + c$

(vi) $-\cos 2x + c$

(vii) $\frac{2}{5} \left(\sin \frac{5x}{2} - \cos \frac{5x}{2} \right) + c$

(viii) $\frac{1}{4}(2x + \sin 2x) + c$

(ix) $-\frac{4}{9} \left[x^{\frac{3}{2}} + (x+3)^{\frac{3}{2}} \right] + c$

(x) $\frac{2}{21} \left[(7x-2)^{\frac{3}{2}} + (7x-5)^{\frac{3}{2}} \right] + c$

(4) $f(x) = \frac{x^2}{2} + \frac{3}{2x^2} + \frac{7}{2}$

EXERCISE 3.2 (A)

I. 1. $\frac{(\log x)^{n+1}}{n+1} + c$ 2. $\frac{2}{5}(\sin^{-1} x)^{\frac{5}{2}} + c$

3. $\log(\operatorname{cosec}(x + \log x) - \cot(x + \log x)) + c$

4. $\frac{-1}{\sqrt{\tan(x^2)}} + c$ 5. $\frac{1}{3}(e^{3x} + 1) + c$

6. $\frac{1}{\log a} \cdot a^{x + \tan^{-1} x} + c$
7. $\frac{1}{2} [\log (\sin e^x)]^2 + c$
8. $\log (e^x - e^{-x}) + c$
9. $\frac{1}{5} \sin^5 x - \frac{1}{7} \sin^7 x + c$
10. $\frac{1}{48} \log (4x^{12} + 5) + c$
11. $\frac{1}{10} \tan x^{10} + c$ 12. $\frac{1}{4} \log (x^4 + 1) + c$
13. $2 \sqrt{\tan x} + c$ 14. $\tan^{-1} x + \frac{1}{x^2 + 1} + c$
15. $\log (3 \cos^2 x + 4 \sin^2 x) + c$
16. $2 \tan^{-1} \sqrt{x} + c$ 17. $\log (10^x + x^{10}) + c$
18. $\frac{\sqrt{1+4x^n}}{2n} + c$
19. $\frac{4}{5} (x+2)^{\frac{5}{2}} - 2 (x+2)^{\frac{3}{2}} + c$
20. $\frac{1}{7} (a^2 + x^2)^{\frac{7}{2}} - \frac{2a^2}{5} (a^2 + x^2)^{\frac{5}{2}} + \frac{a^4}{3} (a^2 + x^2)^{\frac{3}{2}} + c$
21. $-2 \sqrt{2-3x} - \frac{2}{9} (2-3x)^{\frac{3}{2}} + c$
22. $\frac{5}{12} (2x+3)^{\frac{3}{2}} - \frac{11}{2} (2x+3)^{\frac{1}{2}} - \frac{49}{4 \sqrt{2x+3}} + c$
23. $\frac{1}{3} \sin^{-1} \left(\frac{x^3}{3} \right) + c$ 25. $\frac{1}{3} \log \left(\frac{x^3-1}{x^3} \right) + c$
24. $\log (\log (\log x)) + c$
- II.** 1. $2 \cdot \log \left(\sec \frac{x}{2} \right) + c$
2. $\cos a \cdot \log (\sin (x-a)) - (\sin a) x + c$
3. $\cos (a+b) \cdot \log (\sec (x+b)) - (\sin (a+b)) \cdot x + c$
4. $\log (\tan x + 2) + c$
5. $\frac{11}{75} x + \frac{2}{25} \log (3 \sin x + 4 \cos x) + c$
6. $\frac{2x}{13} + \frac{3}{13} \log (2 \cos x + 3 \sin x) + c$
7. $5x - 3 \log |2e^x - 5| + c$
8. $-5x - \log |3e^x - 4| + c$
9. $-x + \frac{7}{8} \log |4e^{2x} - 5| + c$
10. $\frac{\cos^8 x}{8} + \frac{\cos^6 x}{6} + \frac{\cos^4 x}{4} + \frac{\cos^2 x}{2} + \frac{1}{2} \log (\cos^2 x - 1) + c$
11. $\frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} + \log (\sec x) + c$
12. $\sin x - \sin^3 x + \frac{3}{5} \sin^5 x - \frac{1}{7} \sin^7 x + c$
13. $\frac{1}{6} \log \left[\frac{(\sec 3x)^2}{(\sec 2x)^3 (\sec x)^6} \right] + c$
14. $\frac{1}{6} \cos^{11} x - \frac{1}{9} \cos^9 x + \frac{1}{13} \cos^{13} x + c$
15. $-\frac{1}{\log 3} \cdot 3^{\cos^2 x} + c$
16. $\frac{1}{20} \log \left[\frac{\sin^5 4x}{\sin^2 10x} \right] + c$
17. $\frac{1}{2} \log [(1 + \cos^2 x) - \cos^2 x] + c$

EXERCISE 3.2 (B)

- I.** 1. $\frac{1}{4\sqrt{3}} \log \left(\frac{2x - \sqrt{3}}{2x + \sqrt{3}} \right) + c$
2. $\frac{1}{30} \log \left(\frac{5+3x}{5-3x} \right) + c$
3. $\frac{1}{\sqrt{14}} \tan^{-1} \left(\frac{\sqrt{2}x}{\sqrt{7}} \right) + c$
4. $\frac{1}{\sqrt{3}} \log \left(x + \sqrt{x^2 + \frac{8}{3}} \right) + c$

$$5. \frac{1}{2} \sin^{-1} \left(\frac{2x}{\sqrt{11}} \right) + c$$

$$6. \frac{1}{\sqrt{2}} \log \left(x + \sqrt{x^2 - \frac{5}{2}} \right) + c$$

$$7. 9 \sin^{-1} \left(\frac{x}{9} \right) - \sqrt{9 - x^2} + c$$

$$8. 2 \sin^{-1} \left(\frac{x}{2} \right) - \sqrt{4 - x^2} + c$$

$$9. 2 \sin^{-1} \left(\frac{x}{10} \right) - \frac{1}{2} (\sqrt{100 - x^2}) + c$$

$$10. \frac{1}{4} \log \left| \frac{x+2}{x+6} \right| + c$$

$$11. \frac{1}{\sqrt{5}} \log \left(\frac{\sqrt{5} - 1 + 2x}{\sqrt{5} + 1 - 2x} \right) + c$$

$$12. \frac{1}{8\sqrt{2}} \log \left(\frac{2x - 5 - 2\sqrt{2}}{2x - 5 + 2\sqrt{2}} \right) + c$$

$$13. \frac{1}{2\sqrt{19}} \log \left(\frac{3x + 2 + \sqrt{19}}{3x + 2 - \sqrt{19}} \right) + c$$

$$14. \frac{1}{\sqrt{3}} \log \left(x + \frac{5}{6} + \sqrt{x^2 + \frac{5}{3}x + \frac{7}{3}} \right) + c$$

$$15. \log (x + 4 + \sqrt{x^2 - 8x - 20}) + c$$

$$16. \frac{1}{\sqrt{2}} \log \left(x - \frac{3}{4} + \sqrt{x^2 - \frac{3}{2}x + 4} \right) + c$$

$$17. \log \left(x - \frac{1}{2} + \sqrt{x^2 - x - 6} \right) + c$$

$$18. \frac{1}{2\sqrt{7}} \tan^{-1} \left(\frac{2 \tan x}{\sqrt{7}} \right) + c$$

$$19. \frac{1}{\sqrt{2}} \tan^{-1} (\sqrt{2} \tan x) + c$$

$$20. \frac{1}{2\sqrt{3}} \log \left| \frac{\sqrt{3} + \tan x}{\sqrt{3} - \tan x} \right| + c$$

$$\text{II. } 1. \frac{2}{\sqrt{5}} \tan^{-1} \left(\frac{2 \tan \frac{x}{2} + 2}{\sqrt{5}} \right) + c$$

$$2. \frac{1}{3} \log \left[\frac{3 \tan \left(\frac{x}{2} \right) - 1}{3 \tan \left(\frac{x}{2} \right) + 1} \right] + c$$

$$3. \sqrt{2} \tan^{-1} \left(\frac{\tan \frac{x}{2} - 1}{\sqrt{2}} \right) + c$$

$$4. \tan^{-1} \left[2 \tan \left(\frac{x}{2} \right) + 1 \right] + c$$

$$5. \frac{1}{\sqrt{5}} \tan^{-1} (\sqrt{5} \tan x) + c$$

$$6. -\frac{1}{\sqrt{5}} \tan^{-1} \left(\frac{3 \tan x - 2}{\sqrt{5}} \right) + c$$

$$7. \frac{1}{2\sqrt{11}} \log \left(\frac{\sqrt{11} - 2 + \tan x}{\sqrt{11} + 2 - \tan x} \right) + c$$

$$8. \frac{1}{\sqrt{2}} \log \left[\sec \left(x + \frac{\pi}{4} \right) + \tan \left(x + \frac{\pi}{4} \right) \right] + c$$

$$9. \frac{1}{2} \log \left[\sec \left(x + \frac{\pi}{4} \right) + \tan \left(x + \frac{\pi}{4} \right) \right] + c$$

EXERCISE 3.2 (C)

$$\text{I. } 1. \frac{3}{2} \log (x^2 + 6x + 5) - \frac{5}{4} \log \left(\frac{x+1}{x+5} \right) + c$$

$$2. \log (x^2 + 4x - 5) - \frac{1}{2} \log \left(\frac{x-1}{x+5} \right) + c$$

$$3. \frac{1}{2} \log (2x^2 + 3x - 1) + \frac{3}{2\sqrt{17}} \cdot \log \left(\frac{4x + 3 - \sqrt{17}}{4x + 3 + \sqrt{17}} \right) + c$$

$$4. \frac{3}{2} \sqrt{2x^2 + 2x + 1} + \frac{5}{2\sqrt{2}} \cdot \log \left(x + \frac{1}{2} + \sqrt{x^2 + x + \frac{1}{2}} \right) + c$$

$$5. -7 \sqrt{3 + 2x - x^2} + 10 \cdot \sin^{-1} \left(\frac{x-1}{2} \right) + c$$

$$6. \sqrt{x^2 - 16x + 63} +$$

$$\log \left\{ (x-8) + \sqrt{x^2 - 16x + 63} \right\} + c$$

$$7. \sqrt{9x - x^2} + \frac{9}{2} \sin^{-1} \left(\frac{2x-9}{9} \right) + c$$

$$8. \frac{3}{4\sqrt{2}} \log \left(\frac{2\sqrt{2} \sin x + \sqrt{2} - 2}{2\sqrt{2} \sin x + \sqrt{2} + 2} \right) + c$$

$$9. \sqrt{e^{2x} - 1} - \log (e^x + \sqrt{e^{2x} - 1}) + c$$

EXERCISE 3.3

I. 1. $\frac{x^3}{9} (3 \cdot \log x - 1) + c$

2. $-\frac{x^2}{3} \cos 3x + \frac{2}{9} x \sin 3x + \frac{2}{27} \cos 3x + c$

3. $\frac{x^2}{2} \tan^{-1} x - \frac{1}{2} (x - \tan^{-1} x) + c$

4. $\frac{x^3}{3} \tan^{-1} x - \frac{x^2}{6} + \frac{1}{6} \log (1 + x^2) + c$

5. $\frac{1}{4} (\tan^{-1} x) (x^4 - 1) - \frac{x}{12} (x^3 - 3x) + c$

6. $x [(\log x)^2 - 2 (\log x) + 2] + c$

7. $\frac{1}{2} \log (\sec x + \tan x) + \frac{1}{2} \sec x \cdot \tan x + c$

8. $\frac{1}{4} \left[x^2 - x \cdot \sin 2x - \frac{1}{2} \cos 2x \right] + c$

9. $\frac{x^4}{4} \log x - \frac{x^4}{16} + c$

10. $\frac{e^{2x}}{13} [2 \cos 3x + 3 \sin 3x] + c$

11. $\frac{x^2}{2} \sin^{-1} x + \frac{1}{4} x \sqrt{1 - x^2} - \frac{1}{4} \sin^{-1} x + c$

12. $\frac{x^3}{3} \cos^{-1} x - \frac{1}{3} \sqrt{1 - x^2} + \frac{1}{9} (1 - x^2)^{\frac{3}{2}} + c$

13. $(\log x) [\log (\log x) - 1] + c$

14. $-(\sin^{-1} t) \sqrt{1 - t^2} + t + c$

15. $2 [\sqrt{x} \cdot \sin \sqrt{x} + \cos \sqrt{x}] + c$

16. $(\cos \theta) \cdot [1 - \log (\cos \theta)] + c$

17. $\frac{1}{4} \left[\frac{x}{3} \sin 3x + \frac{1}{9} \cos 3x + 3x \sin x \right. \\ \left. + 3 \cos x \right] + c$

18. $-\frac{1}{2} \cos (\log x)^2 + c$

19. $-\frac{1}{2} (\log x)^2 + c$

20. $\frac{x}{6} \sin 3x + \frac{1}{18} \cos x - \frac{1}{14} x \sin 7x \\ - \frac{1}{98} \cos 7x + c$

21. $(3x^{\frac{2}{3}} - 6) \sin \sqrt[3]{x} + 6 \sqrt[3]{x} \cos \sqrt[3]{x} + c$

II. 1. $\frac{e^{2x}}{13} [2 \sin 3x - 3 \cos 3x] + c$

2. $\frac{e^{-x}}{5} [-\cos x + 2 \sin 2x] + c$

3. $\frac{x}{2} [\sin (\log x) - \cos (\log x)] + c$

4. $\sqrt{5} \left[\frac{x}{2} \sqrt{x^2 + \frac{3}{5}} + \right. \\ \left. \frac{3}{10} \log \left(x + \sqrt{x^2 + \frac{3}{5}} \right) \right] + c$

5. $\frac{x^3}{6} \cdot \sqrt{a^2 - x^6} + \frac{a^2}{2} \sin^{-1} \left(\frac{x^3}{a} \right) + c$

6. $\frac{x-5}{2} \sqrt{(x-3)(7-x)} + 2 \sin^{-1} \left(\frac{x-5}{2} \right) + c$

7. $\frac{1}{\log 2} \left\{ \frac{2^x}{2} \sqrt{4^x + 4} + 2 \log (2^x + \sqrt{4^x + 4}) \right\} + c$

8. $\frac{1}{6} (2x^2 + 3)^{\frac{3}{2}} +$

$\sqrt{2} \left[\frac{x}{2} \sqrt{x^2 + \frac{3}{2}} + \frac{3}{4} \log \left(x + \sqrt{x^2 + \frac{3}{2}} \right) \right] + c$

$$9. -\frac{1}{3}(5-4x-x^2)^{\frac{3}{2}} - (x+2)\sqrt{5-4x-x^2} - 9\sin^{-1}\left(\frac{x+2}{3}\right) + c$$

$$10. \frac{(1+2\tan x)}{4}\sqrt{\tan^2 x + \tan x - 7} - \frac{29}{8}\log\left\{\frac{1}{2} + \tan x + \sqrt{\tan^2 x + \tan x - 7}\right\} + c$$

$$11. \left(\frac{x+1}{2}\right)\sqrt{x^2+2x+5} + 2\log\left\{x+1+\sqrt{x^2+2x+5}\right\} + c$$

$$12. \sqrt{2}\left\{\left(\frac{4x+3}{8}\right)\sqrt{x^2+\frac{3}{2}x+2} + \frac{23}{16\sqrt{2}}\log\left[\left(x+\frac{3}{4}\right)+\sqrt{x^2+\frac{3}{2}x+2}\right]\right\} + c$$

III. 1. $e^x(2+\cot x) + c$ 2. $e^x \cdot \tan \frac{x}{2} + c$ 3. $e^x \cdot \frac{1}{x} + c$ 4. $e^x \cdot \left(\frac{1}{x+1}\right) + c$

5. $e^x \cdot (\log x)^2 + c$ 6. $e^{5x} \cdot \log x + c$ 7. $e^{\sin^{-1} x} \cdot x + c$

8. $\frac{(1+x)^2}{2}\left(\log(1+x) - \frac{1}{2}\right) + c$ 9. $x \cdot \operatorname{cosec}(\log x) + c$

EXERCISE 3.4

I. 1. $\frac{1}{4}\log(x-1) - 2\log(x+2) + \frac{11}{4}(x+3) + c$

2. $\frac{1}{6}\tan^{-1}x + \frac{1}{15\sqrt{2}}\log\left(\frac{x-\sqrt{2}}{x+\sqrt{2}}\right) - \frac{\sqrt{3}}{10}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + c$

3. $\frac{51}{41}\log(2x+9) + \frac{31}{41}\log(3x-7) + c$ 4. $-\frac{8}{5}\log(x+4) - \frac{2}{5}\log(x-1) + c$

5. $x - \log(x+3) + \log(x-2) + c$ 6. $x^2 + 3x + \frac{5}{3}\log(3x+1) + \log(x-1) + c$

7. $\frac{1}{2}\log\left|\frac{2x+1}{2x-1}\right| + 3\log(x+3) + c$ 8. $\frac{1}{5}\log\left(\frac{x^5}{x^5+1}\right) + c$ 9. $\frac{11}{\sqrt{5}}\tan^{-1}\left(\frac{x}{2}\right) - \frac{9}{2}\tan^{-1}\left(\frac{x}{2}\right) + c$

10. $2\log\left(\frac{x+1}{x-1}\right) + \frac{5}{2\sqrt{2}}\log\left(\frac{x+\sqrt{2}}{x-\sqrt{2}}\right) + c$ 11. $\log\left(\frac{2+x^2}{3+x^2}\right) + c$

12. $\frac{1}{5\log 2}\log\left(\frac{2^x-4}{2^x+1}\right) + c$ 13. $\frac{5}{2}\left(\frac{1}{x+1}\right) + \frac{11}{4}\log\left(\frac{x+1}{x+3}\right) + c$

14. $6\log x - \log(x+1) - \frac{9}{x+1} + c$ 15. $\frac{1}{8}\log\left(\frac{x^6(x^3+3)}{(3x^3+1)^3}\right) + c$

16. $\frac{1}{3}\log(x-1) - \frac{1}{6}\log(x^2+x+1) - \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + c$

$$17. 3 \cdot \log(\sin x - 2) - \frac{4}{\sin x - 2} + c$$

$$18. \frac{1}{2} \log(\cos x + 1) + \frac{1}{6} \log(\cos x - 1) - \frac{2}{3} \log(2 \cos x + 1) + c$$

$$19. \frac{1}{8} \log\left(\frac{\cos x - 1}{\cos x + 1}\right) + \frac{1}{4 \cdot (\cos x + 1)} + c$$

$$20. \frac{1}{6} \log\left[\frac{(1 + 2 \sin x)^4}{(1 - \sin x)(1 + \sin x)^3}\right] + c$$

$$21. \frac{1}{10} \log(1 - \cos x) - \frac{1}{2} \log(1 + \cos x) + \frac{2}{5} \log(3 + 2 \cos x) + c$$

$$22. \frac{1}{2} \log\left[\frac{e^x + 1}{(e^{2x} + 9)^{\frac{1}{2}}}\right] + \frac{1}{6} \tan^{-1}\left(\frac{e^x}{3}\right) + c$$

$$23. \frac{5}{26} \log\left[\frac{(3 \log x + 2)^2}{\sqrt{(\log x)^2 + 1}}\right] + \frac{11}{26} \tan^{-1}(\log x) + c$$

MISCELLANEOUS EXERCISE 3

(I)

1	2	3	4	5	6	7	8	9	10
B	A	B	A	D	B	A	A	C	B
11	12	13	14	15	16	17	18	19	20
A	A	D	C	A	D	A	D	C	A

(II) (1) $\frac{2}{7} x^{\frac{7}{2}} - \frac{8}{5} x^{\frac{5}{2}} - \frac{8}{3} x^{\frac{3}{2}} + c$

(2) $\frac{x^7}{7} - \frac{x^6}{6} + \frac{x^5}{5} - \frac{x^4}{4} + \frac{x^3}{3} - \frac{x^2}{2} + x - \log(x + 1) + c$

(3) $\frac{1}{15} (6x + 5)^{\frac{5}{2}} + c$

(4) $\frac{t^2}{2} - 2t + 3 \cdot \log(t + 1) + \frac{1}{t + 1} + c$

(5) $3 \tan x - 2 \sec x + c$

(6) $\tan \theta - \cot \theta - 3 \theta + c$

(7) $\frac{1}{48} (2 \sin 6x + 3 \sin 4x + 6 \sin 2x + 12 x) + c$

(8) $\frac{1}{2} \sin 2x - \frac{1}{3} \sin 3x + c$

(9) $\frac{\pi}{4} x - \frac{1}{4} x^2 + c$

(III) (1) $\frac{1}{4} (1 + \log x)^4 + c$

(2) $(\tan^{-1} x) x - \frac{1}{2} \log(1 + x^2) - (1 - x) \tan^{-1}(1 - x) + \frac{1}{2} \log(x^2 - 2x + 2) + c$

(3) $-\cot(\log x) + c$

(4) $\frac{2}{3} \sec x^{\frac{3}{2}} + c$

- (5) $x \log (1 + \cos x) + c$ (6) $\frac{1}{3} \sin^{-1} (x^3) + c$
- (7) $\frac{1}{4} \log (3 - 2 \cot x) + c$ (8) $x \cdot \left(\log (\log x) - \frac{1}{\log x} \right) + c$
- (9) $\frac{2}{\sqrt{13}} \tan^{-1} \left(\frac{2 \tan \left(\frac{x}{2} \right) - 3}{\sqrt{13}} \right) + c$ (10) $\frac{1}{4} \left(2 \sec^{-1} x + \frac{2\sqrt{x^2 - 1}}{x^2} \right) + c$
- (11) $-\frac{3}{2} \sqrt{-2x^2 + x + 3} + \frac{7}{4\sqrt{2}} \sin^{-1} \left(\frac{2x - 1}{\sqrt{7}} \right) + c$ (12) $x \cdot \log (x^2 + 1) - 2 [x - \tan^{-1} x] + c$
- (13) $\frac{1}{4} e^{2x} \cdot [\sin 2x - \cos 2x] + c$
- (14) $\frac{1}{18} \log (3x - 1) + \frac{1}{2} \log (x - 1) - \frac{4}{9} \log (3x - 2) + c$
- (15) $\frac{1}{6} \log \left\{ \frac{(\cos x - 1)(\cos x + 1)^3}{(2 \cos x + 1)^4} \right\} + c$
- (16) $\left(\frac{\tan x - 1}{2} \right) \sqrt{7 + 2 \tan x - \tan^2 x} + 4 \sin^{-1} \left(\frac{\tan x - 1}{2\sqrt{2}} \right) + c$
- (17) $\frac{1}{4} \log \left\{ \frac{(x - 1)^3 (x + 3)}{(x + 1)^4} \right\} + c$ (18) $\frac{1}{5} \log \left(\frac{x^5}{x^5 + 1} \right) + c$
- (19) $2 \sqrt{\tan x} + c$ (20) $\frac{1}{3 \cot^3 x} + \frac{2}{\cot x} - \cot x + c$

4. DEFINITE INTEGRATION

EXERCISE 4.1

- I. (1) 4 (2) $\frac{64}{3}$ (3) $e^2 - 1$ (4) 6 (5) 20

EXERCISE 4.2

- I. (1) $\frac{64}{3}$ (2) $\log \left(\frac{25}{24} \right)$ (3) $-\left(1 + \frac{\pi}{4} \right)$ (4) 2
- (5) $\frac{1}{18} [13\sqrt{13} + 7\sqrt{7} - 3\sqrt{3} - 27]$ (6) $1 - \frac{3\pi}{4}$ (7) $\frac{4}{7\sqrt{2}}$ (8) 1 (9) $\frac{3\pi}{16}$
- (10) $\frac{1}{3} \left(\tan^{-1} \frac{4}{3} + \tan^{-1} \frac{2}{3} \right)$ (11) π (12) $\frac{\pi}{6}$ (13) 1 (14) $\frac{\pi}{4} - \frac{1}{2}$ (15) 1

- II. (1) $\frac{\pi}{4} - \frac{1}{2} \log 2$ (2) $\frac{1}{2} \log 2$ (3) $\frac{\pi}{4}$
 (4) 0 (5) $\frac{2}{3} \tan^{-1} \left(\frac{1}{3} \right)$ (6) $\frac{1}{4} \log \left(\frac{2\sqrt{2}+1}{2\sqrt{2}-1} \right)$
 (7) $\log \left(\frac{4}{3} \right)$ (8) $\frac{1}{ab} \left[\tan^{-1} \left(\frac{ae}{b} \right) - \tan^{-1} \left(\frac{a}{be} \right) \right]$
 (9) $\frac{\pi}{4}$ (10) $\frac{4}{3}$ (11) $\frac{\pi}{2} - 1$
 (12) $\frac{8}{3}$ (13) $\frac{\pi}{2} - 1$
 (14) $e^{\frac{\pi}{4}} \left[\frac{\pi}{4} + 1 \right] - \left[\frac{\pi}{2} + 1 \right]$ (15) $\sin(\log 3)$
- III. (1) $\frac{\pi}{4}$ (2) 0 (3) 0 (4) 0 (5) $\frac{16}{77}(3)^{\frac{7}{2}}$
 (6) $\frac{8}{21}$ (7) 0 (8) $\frac{\pi^2}{6\sqrt{3}}$ (9) 0 (10) 0
 (11) $4 \log \left(\frac{1+\sqrt{5}}{2} \right)$ (12) 0 (13) $\frac{16}{105}$ (14) $\frac{\pi}{3}$ (15) $\frac{\pi}{2} \log \left(\frac{1}{2} \right)$

MISCELLANEOUS EXERCISE 4

(I)

1	2	3	4	5	6	7	8	9	10
A	A	C	C	D	C	A	D	B	A

- (II) (1) $\frac{1}{10}(3 - \log 3)$ (2) $2 - \sqrt{2}$ (3) $6 - 4 \log 2$ (4) $\frac{1}{8}$ (5) $\frac{1}{21}$
 (6) $\pi - 2$ (7) $\frac{1}{3} \log 2$ (8) $\frac{\pi}{5}$ (9) 0 (10) $\frac{\pi}{2}$
- (III) (1) $\frac{\pi^2}{16}$ (2) $\frac{2}{\sqrt{35}} \tan^{-1} \sqrt{\frac{7}{5}}$ (3) $\frac{1}{\sqrt{5}a} \log \left(\frac{7+3\sqrt{5}}{2} \right)$ (4) $\frac{\pi}{20}$
 (5) $\frac{\pi}{2} - \log 2$ (6) $\frac{1}{2} \left(\frac{\pi}{4} - \log \sqrt{2} \right)$ (7) $-\frac{\pi}{2} \log 2$ (8) $\frac{\pi^3}{6}$
 (9) $\log \left(\frac{5+3\sqrt{3}}{1+\sqrt{3}} \right)$ (10) $\frac{17}{2}$
- (IV) (1) $\frac{1}{2}$ when $a = 0$; $\frac{9}{2}$ when $a = 4$ (2) $k = \frac{1}{2}$

5. APPLICATION OF DEFINITE INTEGRAL

EXERCISE 5.1

- (1) (i) 25 (ii) 16 (iii) 20
 (iv) 1 (v) $2 \log 4$ (vi) $\frac{32}{3}$
 (vii) $\frac{128}{3}$ sq. units
- (2) (i) $\frac{128}{3}$ (ii) $\frac{16}{3}$
 (3) (i) $\frac{1}{12}$ (ii) $\frac{8}{3}$ (iii) $\frac{32}{3}$
 (iv) $8\frac{a^2}{3}$ (v) $\frac{1}{6}$

MISCELLANEOUS EXERCISE 5

(I)

1	2	3	4	5	6	7	8	9	10
A	A	C	B	A	D	B	D	A	B

11	12	13	14	15	16	17	18	19	20
A	D	B	B	C	C	A	D	A	C

(II)

1. (i) 10 (ii) 2 (iii) $\frac{1}{2}$ 5. $\frac{\pi}{3}$ 6. $\frac{1}{6}$ 7. $\frac{\pi}{4} - \frac{1}{2}$
 2. 9π 3. 20π 8. $\frac{56}{3}$ 9. $36\frac{3}{4}$ 10. $\frac{7}{3}$
 4. (i) $\frac{16}{3}$ (ii) $\frac{8}{3}$ (iii) $\frac{1}{3}$

6. DIFFERENTIAL EQUATIONS

EXERCISE 6.1

- (1) (i) 2, 1 (ii) 2, 3 (iii) 1, 2 (iii) $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$ (iv) $8 \left(\frac{dy}{dx} \right)^3 - 27y = 0$
 (iv) 3, 1 (v) 2, 1 (vi) 3, 2 (v) $\frac{d^2y}{dx^2} - 25y = 0$ (vi) $2 \frac{d^2y}{dx^2} + \left(\frac{dy}{dx} \right)^3 = 0$
 (vii) 2, not defined (viii) 2, 2 (vii) $(x^2 + xy) \frac{dy}{dx} + y = 0$ (viii) $\frac{d^2y}{dx^2} - 7 \frac{dy}{dx} + 10y = 0$
 (ix) 3, 3 (x) 2, 1

EXERCISE 6.2

- (1) (i) $2x^3 + 3xy^2 \frac{dy}{dx} - y^3 = 0$ (ix) $xy \frac{d^2y}{dx^2} + x \left(\frac{dy}{dx} \right)^2 - 2y \frac{dy}{dx} = 0$
 (ii) $xy \frac{d^2y}{dx^2} + x \left(\frac{dy}{dx} \right)^2 - y \frac{dy}{dx} = 0$ (x) $\frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + 5y = 0$

$$(2) \quad y = \frac{dy}{dx} (x-a) \quad (3) \quad 2a \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 0$$

$$(4) \quad x + 4y \frac{dy}{dx} = 0 \quad (5) \quad 3 \frac{dy}{dx} + 2 = 0$$

$$(6) \quad 81 \left(\frac{d^2y}{dx^2}\right)^2 = \left[\left(\frac{dy}{dx}\right)^2 + 1\right]^3$$

$$(7) \quad y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$$

EXERCISE 6.3

$$(2) \quad (i) \quad \tan^{-1} y = \tan^{-1} x + c$$

$$(ii) \quad 2e^{-3y} + 3e^{2x} = c \quad (iii) \quad x = cy$$

$$(iv) \quad \tan x \cdot \tan y = c \quad (v) \quad \sin y \cdot \cos x = c$$

$$(vi) \quad y = -kx + c$$

$$(vii) \quad 2(x^2 + y^2) + 2(x \sin 2x + y \sin 2y) + \cos 2y + \cos 2x + c = 0$$

$$(viii) \quad 2y^2 \tan^{-1} x + 1 = cy^2$$

$$(ix) \quad 4e^x + 3e^{-2y} = c$$

$$(x) \quad 3e^x + 3e^{-y} + x^3 = c$$

$$(3) \quad (i) \quad (1 + e^x)^3 \tan y = 0$$

$$(ii) \quad (1 + x^2)(1 - y^2) = 5$$

$$(iii) \quad y = ex \log x \quad (iv) \quad (\sin x)(e^y + 1) = \sqrt{2}$$

$$(v) \quad 2(2 + e^y) = 3(x + 1)$$

$$(vi) \quad \cos\left(\frac{y-2}{x}\right) = a$$

$$(4) \quad (i) \quad \tan\left(\frac{x+y}{2}\right) = x + c$$

$$(ii) \quad c + 2y = a \log\left(\frac{x-y-a}{x-y+a}\right)$$

$$(iii) \quad \sin(x^2 + y^2) + 2x = c$$

$$(iv) \quad x = \tan(x - 2y) + c$$

$$(v) \quad (2x - y) - \log(x - y + 2) + 1 = 0$$

EXERCISE 6.4

$$(1) \quad \cos\left(\frac{y}{x}\right) dy = \log(x) + c$$

$$(2) \quad x^2 - y^2 = cx \quad (3) \quad x + 2ye^{\frac{x}{y}} = c$$

$$(4) \quad xy^2 = c^2(x + 2y) \quad (5) \quad x^2 + y^2 = cx$$

$$(6) \quad y = c(x + y)^3 + x$$

$$(7) \quad x \left[1 - \cos\left(\frac{y}{x}\right)\right] = \sin\left(\frac{y}{x}\right)$$

$$(8) \quad x + ye^{\frac{x}{y}} = c \quad (9) \quad \log(y) + \frac{y}{x} = c$$

$$(10) \quad x^2y = 4 \quad (11) \quad x^2 + y^2 = x^4$$

$$(12) \quad \tan^{-1}\left(\frac{y}{x}\right) = \log(x) + c$$

$$(13) \quad (3x + y)^3(x + y)^2 = c$$

$$(14) \quad c = \log(x) + \frac{x}{x+y} \quad (15) \quad x^2 - y^2 = cx$$

EXERCISE 6.5

$$1. \quad (i) \quad \frac{x^5}{5} - \frac{3x^2}{2} - xy = c$$

$$(ii) \quad ye^{\tan x} = e^{\tan x}(\tan x - 1) + c$$

$$(iii) \quad x = y(c + y^2)$$

$$(iv) \quad y(\sec x + \tan x) = \sec x + \tan x - x + c$$

$$(v) \quad x^2y = \frac{x^4 \log x}{4} - \frac{x^4}{16} + c$$

$$(vi) \quad x + y + 1 = ce^y$$

$$(vii) \quad 2y = (x + a)^5 + 2c(x + a)^3$$

$$(viii) \quad r \sin^2 \theta + \frac{\sin^4 \theta}{2} = c$$

$$(ix) \quad \frac{y^3}{3} = xy + c$$

$$(x) \quad y = \sqrt{1 - x^2} + c(1 - x^2)$$

$$(xi) \quad y = \frac{1}{2} e^{\tan^{-1} x} + c e^{-\tan^{-1} x}$$

$$2. \quad 3(x + 3y) = 2(1 - e^{3x})$$

$$4. \quad y = 4 - x - 2e^x$$

$$3. \quad 4x^2 + 9y^2 = 36$$

$$5. \quad 1 + y = 2e^{\frac{x^2}{2}}$$

EXERCISE 6.6

$$1. \quad 8 \text{ times of original.}$$

$$2. \quad 95.4 \text{ years}$$

$$3. \quad 36.36^\circ\text{C}$$

$$4. \quad 5656$$

$$5. \quad \frac{\log 3}{k}$$

$$6. \quad \frac{27}{5} \text{ gms}$$

$$7. \quad (3000) \left(\frac{4}{9} \right)^{\frac{t}{40}}$$

$$8. \quad 1 \text{ hour}$$

$$10. \quad r = 3 - t$$

$$11. \quad 27,182$$

$$12. \quad \left(10 - \frac{p}{10} \right)^2 \%$$

MISCELLANEOUS EXERCISE 6

(I)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
D	A	C	B	A	D	C	B	C	D	B	A	B	B	B

(II) (1) (i) 2, 1 (ii) 3, 10 (iii) 2, 3 (iv) 1.4 (v) 4, not defined

(3) (i) $xy \frac{d^2y}{dx^2} + x \left(\frac{dy}{dx} \right)^2 - 2y \frac{dy}{dx} = 0$ (ii) $\frac{d^2y}{dx^2} + y = 0$ (iii) $(y - a) \frac{d^2y}{dx^2} + \left(\frac{dy}{dx} \right)^2 = 0$

(iv) $2x^2y \frac{d^2y}{dx^2} + 2x^2 \left(\frac{dy}{dx} \right)^2 + y = 0$ (v) $\frac{d^2y}{dx^2} - 9y = 0$

(4) (i) $2xy \frac{dy}{dx} + x^2 - y^2 = 0$ (ii) $2b \frac{d^2y}{dx^2} - 1 = 0$ (iii) $x + 4y \frac{dy}{dx} = 0$ (iv) $2 \frac{dy}{dx} - 3 = 0$

(v) $4y \frac{dy}{dx} - 9x = 0$

(5) (i) $2e^{-3y} + 3e^{2x} + 6c = 0$ (ii) $\log(y) = \frac{x^3}{3} + x + c$ (iii) $y = \frac{x}{2} \log(x^2) + 2 + cx$

(iv) $y = 1 + x \log x + cx$ (v) $y = x^2 + c \cdot \operatorname{cosec} x$ (vi) $x \log y = (\log y)^2 + c$

(vii) $4xe^{2y} + 5e^{-y} = c$

(6) (i) $ex \log x - y = 0$ (ii) $x = 2y^2$ (iii) $y \operatorname{cosec}^2 x + 2 = 4 \sin 2x$

(iv) $\log \sqrt{x^2 + y^2} + \tan^{-1} \left(\frac{y}{x} \right) = \frac{\pi}{4}$ (v) $x + 2ye^{\frac{x}{y}} = 2$

(8) $x^2 + y^2 = 4x + 5$ (9) $r = (63t + 27)^{\frac{1}{3}}$ (10) $\frac{20}{9} \text{ years}$

7. PROBABILITY DISTRIBUTIONS

EXERCISE 7.1

1. $\{-6, -4, -2, 0, 2, 4, 6\}$

2. $\{0, 1, 2\}$

3. (i) p.m.f. (ii) Not p.m.f

(iii) p.m.f (iv) Not p.m.f

(v) Not p.m.f. (iv) p.m.f

5.

X	0	1	2
$P(X)$	$\frac{2}{3}$	$\frac{2}{9}$	$\frac{1}{9}$

6.

X	0	1	2	3	4
$P(X)$	$\left(\frac{4}{5}\right)^4$	$\left(\frac{4}{5}\right)^3 \frac{1}{5}$	$\left(\frac{4}{5}\right)^2 \left(\frac{1}{5}\right)^2$	$\frac{4}{5} \left(\frac{1}{5}\right)^3$	$\left(\frac{1}{5}\right)^4$

7.

X	0	1	2
$P(X)$	$\frac{9}{16}$	$\frac{3}{8}$	$\frac{1}{16}$

4. (i)

X	0	1	2
$P(X)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

(ii)

X	0	1	2	3
$P(X)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

(iii)

X	0	1	2	3	4
$P(X)$	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

8. (i) $\frac{1}{10}$ (ii) $\frac{3}{10}$ (iii) $\frac{1}{5}$

9. $-0.05, 2.2475$ 10. $\frac{7}{3}, \frac{524}{54}$ 11. 1.5

12. $\frac{1}{3}$ 13. 4.67 14. 2.41

15. $17.53, 4.9, 2.21$ 16. $0.7, 0.21$

EXERCISE 7.2

1. (i) p.d.f. (ii) Not a p.d.f

(iii) Not a p.d.f

2. (a) $\frac{2.25}{16}$, (b) $\frac{3}{16}$, (c) $\frac{3}{4}$

3. (i) p.d.f. (ii) $\frac{1}{9}$ (iii) $\frac{1}{9}$

4. (i) $\frac{1}{2}, \frac{35}{64}$ (ii) $6, \frac{11}{32}, \frac{1}{2}$ 8. (i) $\frac{x^2}{16}$ (ii) $\frac{1}{64}, 0.18, 1$
5. (i) $\frac{1}{4}$ (ii) $\frac{1}{2}$ (iii) $\frac{7}{16}$ 9. $\frac{2}{9}, 0, \frac{8}{9}, \frac{7}{9}$
6. (i) $\frac{2}{5}$ (ii) $\frac{1}{5}$ 10. $\frac{1}{\log 3}, \frac{4}{\log 3}, \frac{4(\log 3 - 1)}{(\log 3)^2}$
7. (i) $\frac{1}{2}$ (ii) $\frac{11}{16}$ (iii) 0.6328

MISCELLANEOUS EXERCISE 7

(I)

1	2	3	4	5	6	7	8	9	10
B	C	A	B	C	B	B	A	D	B

(II) Solve the following :

- (1) (i) Discrete $\{1, 2, 3, \dots, 100000\}$ (ii) Continuous. (iii) Continuous.
 (iv) Discrete $\{0, 1, 2, 3, 4, 5\}$ (v) Continuous
- (2) (i) $\frac{1}{21}$ (ii) $\frac{10}{21}, \frac{1}{7}, \frac{6}{7}$ (3) (i) 0.5 (ii) 0.7 (iii) 0.55 (iv) 0.45
- (5) $\frac{1}{4}$

x	1	2	3	4	5
$P(X)$	$\frac{1}{20}$	$\frac{3}{20}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{20}$
$F(x)$	$\frac{1}{20}$	$\frac{1}{5}$	$\frac{9}{20}$	$\frac{19}{20}$	1

(6)

X	0	1	2	3	4
$P(X)$	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

$$\frac{{}^4C_x}{2^4}$$

(7) (i)

X	0	1	2
$P(X)$	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

(ii)

X	0	1	2
$P(X)$	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

(8) (i) $\frac{1}{10}$ (ii) $\frac{17}{100}$ (iii) $\frac{3}{10}$

(9)

X	-3	-2	-1	0	1	2	3	4
$F(X)$	0.1	0.3	0.5	0.65	0.75	0.85	0.9	1
$P(x)$	0.1	0.2	0.2	0.15	0.10	0.10	0.05	0.10

(i) 0.55 (ii) 0.25

(10) (i) $\frac{11}{5}, \frac{14}{25}, \frac{\sqrt{14}}{5}$ (ii) $\frac{1}{5}, \frac{14}{25}, \frac{\sqrt{14}}{5}$ (iii) $\frac{n+1}{2}, \frac{n^2-1}{12}, \sqrt{\frac{n^2-1}{12}}$ (iv) $\frac{5}{2}, \frac{5}{4}, \frac{\sqrt{5}}{2}$

(11) ₹ 5.5, 8.25 (12) 0, 1 (13) (i) $\frac{1}{2}$ (ii) $\frac{11}{16}$ (iii) $\frac{81}{128}$

(15) $k = \frac{1}{\theta}, \frac{1}{e}$ (16) $k = \frac{1}{4}, F(x) = \frac{\sqrt{x}}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}$

8. BINOMIAL DISTRIBUTION

EXERCISE 8.1

- (i) $\frac{3}{2^5}$ (ii) $\frac{7}{2^6}$ (iii) $\frac{63}{64}$
- $\frac{25}{216}$ 3. $29 \left(\frac{19^9}{20^{10}} \right)$
- (i) $\frac{1}{1024}$ (ii) $\frac{45}{1024}$
- (i) $(0.95)^5$ (ii) $(1.2)(0.95)^4$
- (iii) $1 - (1.2)(0.95)^4$ (iv) $1 - (0.95)^5$
- $\left(\frac{9}{10} \right)^4$ 7. $\frac{11}{243}$
- (i) $1 - \left(\frac{99}{100} \right)^{50}$ (ii) $50 \left(\frac{99^{49}}{100^{50}} \right)$
- (iii) $1 - 149 \left(\frac{99^{49}}{100^{50}} \right)$
- (i) $\frac{1}{20^3}$ (ii) $3 \left(\frac{19}{20^3} \right)$ (iii) $3 \left(\frac{19^2}{20^3} \right)$ (iv) $\left(\frac{19}{20} \right)^3$
- $\frac{7}{3} \left(\frac{5}{6} \right)^5$ 11. $22 \left(\frac{9^3}{10^{11}} \right)$
- (i) 4, 2.4 (ii) 10, 2.4 (iii) $\frac{2}{5}; \sqrt{6}$ (iv) $\frac{8}{5}$

MISCELLANEOUS EXERCISE 8

(I)

1	2	3	4	5	6	7
B	D	D	C	B	C	B

(II) Solve the following :

(1) (i) $2 \times (0.8)^9$ (ii) $1 - (0.8)^{10}$
(iii) $1 - (8 \cdot 2)(0.2)^9$

(2) (i) $p = \frac{1}{2}$, $\text{Var}(X) = 2.5$
(ii) $n = 10$, $p = \frac{1}{2}$

(3) (i) $\frac{63}{256}$ (ii) $\frac{105}{512}$

$$(4) \quad 45 \left(\frac{2^{26}}{2^{10}} \right)$$

(5) (i) $0.65 \times (0.95)^{16}$
(ii) $(2.0325) \times (0.95)^{14}$
(iii) $1 - (1.6) \times (0.95)^{16}$

(6) 0.2114 (7) $1.4 \times (0.9)^4$

$$(8) \quad 6.97 \times (0.97)^{19}$$

(9) 0.3456 (10) $\frac{30.44}{5^8}$

(11) (i) $(0.998)^8$ (ii) $1.014 \times (0.998)^7$

(iii) $1 - 1.014 \times (0.998)^7$

$$(12) \quad 775.44 \times (0.003)^{38}$$

(13) (i) 0.9^{10} (ii) 0.9^9

(iii) $0.45 \times (0.9)^8$

$$\text{(iv) } 1 - 2 \cdot 16 \times (0 \cdot 9)^8$$

$$(14) \quad (i) \quad \frac{1}{5^4}, \frac{16}{5^4}, \frac{96}{5^4}, \frac{256}{5^4}, \frac{256}{5^4}$$

(ii) (a) $\frac{608}{5^4}$ (b) $1 - \frac{33}{5^8}$

(15) (i) $35 \times 8 \times \frac{81}{5^7}$ (ii) $1 - \frac{12393}{5^7}$

$$(16) \quad (i) \quad \frac{\log 0.5}{\log 0.99}$$

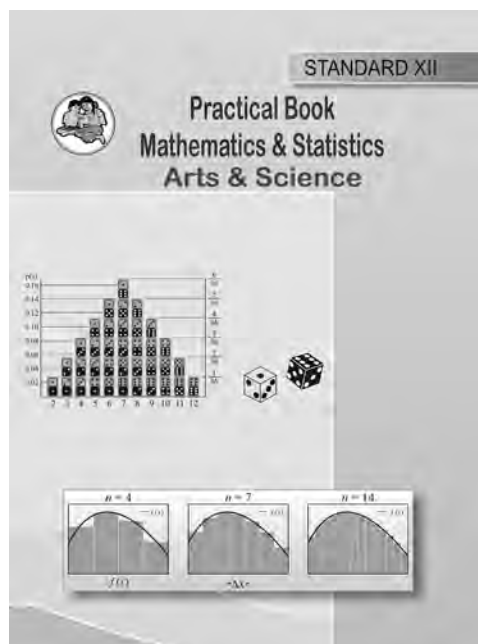
$$(17) \quad \frac{1}{5}$$



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